习题答案 2.3

1.

（1）不是，

（2）是　$x^2/2+2xy-y^2/2=c$

（3）是　$ax^2/2+bxy+cy^2/2=C$

（4）是　$(t^2+1)\sin u=c$

(5）不是

(6）　是　$x^3/3+y\ln(x)-y^2=c$

(7）$c=2b$，是，$ax^3/3+bxy^2=C$；否则不是

(8）是，$(s^2-s)/t=c$

(9) 是　$\int f(x^2+y^2)d(x^2+y^2)=c$

2.

（1）$\mu=x$

（2）$\mu=y^{-4}$，

（3）$\mu=x^3$，

（4）$\mu=y^{-n}\exp\left(\int(n-1)p(x)dx\right)$

3.

（1）$y=(x^2+c)e^{-x^2}$，

(2）$y=-\dfrac{x}{3}+\dfrac{c}{x^2}$，

（3）$x\sin(x+y)=c$

（4）$x^4y^2+x^3y^5=c$，

（5）$e^y x\cos x+e^y(y-1)\sin x=c$

4. $f(x)=\dfrac{x}{2}$，

5。$a=1$, 方程解是 $x+e^{-x}\sin y=c$

6.

（1）$\mu=x^3y^2$，　$x^5y^4+2x^4y^3=c$

（2）$\mu = x^{-3}$， 方程解是：$\frac{3}{2}t^2 - \frac{1}{2}x^{-2} + \frac{t}{x} = c$

（3） $\mu =$ x＋1，方程解是：$x^3y + x^2y^2 + 2x^2y + 2xy^2 + xy + y^2 = c$

（4）$\mu = x^{-2}$， 方程解是$\frac{1}{3}x^3y^3 - \frac{y}{x} = c$

7. 显然只需要证明 f(xy)/ $\varphi(x,y)$ 对 y 的偏导等于 g(xy)/ $\varphi(x,y)$ 对 x 的偏导

$\frac{\partial(\mathrm{f(xy)}\phi^{-1}(x,y))}{\partial y} = \frac{-f'(x,y)g(xy)+f(xy)g'(xy)}{(f(xy)-g(xy))^2}$,同样我们得到：

$\frac{\partial(g(\mathrm{xy})\phi^{-1}(x,y)) =}{\partial x}\frac{-f'(xy)g(xy)+f(xy)g'(xy)}{(f(xy)-g(xy))^2}$，则$\varphi(x,y)$是其积分因子。

8. $\frac{M_y - N_x}{N-M} = f(x+y)$；

$\frac{M_y - N_x}{yN - xM} = g(xy)$

9．齐次（0 次）函数的特点是：$xM_x + yM_y = 0,\quad xN_x + yN_y = 0$

$$\frac{\partial(\mu(x,y)M(x,y))}{\partial y} = \frac{-MN + yN\frac{\partial M}{\partial y} - yM\frac{\partial N}{\partial y}}{(xM+yN)^2}$$

$$\frac{\partial(\mu(x,y)N(x,y))}{\partial x} = \frac{-MN + xM\frac{\partial N}{\partial x} - xN\frac{\partial M}{\partial x}}{(xM+yN)^2}$$

验证让二者相等：

$$\frac{\partial(\mu(x,y)M(x,y))}{\partial y} - \frac{\partial(\mu(x,y)N(x,y))}{\partial x}$$

$$= \frac{N\left(y\frac{\partial M}{\partial y} + x\frac{\partial M}{\partial x}\right) - M\left(y\frac{\partial N}{\partial y} + x\frac{\partial N}{\partial x}\right)}{(xM+yN)^2} = 0$$

10.证明：因为 $\mu(x,y)G(F(x,y))(Mdx+Ndy) = G(F(x,y))dF = d(\int G(F(x,y))dF)$

所以 $\mu(x,y)G(x,y)$ 也是（2.3.2）的一个积分因子

11. 有积分因子的定义知

$$\frac{\partial \mu_1}{\partial y}M+\mu_1\frac{\partial M}{\partial y}=\frac{\partial \mu_1}{\partial x}N+\mu_1\frac{\partial N}{\partial x}$$

$$\frac{\partial \mu_2}{\partial y}M+\mu_2\frac{\partial M}{\partial y}=\frac{\partial \mu_2}{\partial x}N+\mu_2\frac{\partial N}{\partial x}$$

由此得 $M\left(\mu_2\frac{\partial \mu_1}{\partial y}-\mu_1\frac{\partial \mu_2}{\partial y}\right)=N\left(\mu_2\frac{\partial \mu_1}{\partial x}-\mu_1\frac{\partial \mu_2}{\partial x}\right)$

记 $F=\frac{\mu_1}{\mu_2}$，则有

$$\frac{\partial F}{\partial x}=\frac{\mu_2\frac{\partial \mu_1}{\partial x}-\mu_1\frac{\partial \mu_2}{\partial x}}{\mu_2^2}$$

$$\frac{\partial F}{\partial y}=\frac{\mu_2\frac{\partial \mu_1}{\partial y}-\mu_1\frac{\partial \mu_2}{\partial y}}{\mu_2^2}$$

于是

$$dF=\frac{\partial F}{\partial x}dx+\frac{\partial F}{\partial y}dx=\frac{\mu_2\frac{\partial \mu_1}{\partial x}-\mu_1\frac{\partial \mu_2}{\partial x}}{\mu_2^2}dx+\frac{\mu_2\frac{\partial \mu_1}{\partial y}-\mu_1\frac{\partial \mu_2}{\partial y}}{\mu_2^2}dy$$

$$=\frac{\mu_2\frac{\partial \mu_1}{\partial x}-\mu_1\frac{\partial \mu_2}{\partial x}}{\mu_2^2}\left(dx+\frac{N}{M}dy\right)=0$$

所以，F=c 是通解