答案 5.3

1.

1) 鞍点
2) 不稳定结点
3) 鞍点
4) 稳定的退化结点
5) 稳定焦点
6) 中心点
7) 不稳定焦点
8) 稳定结点

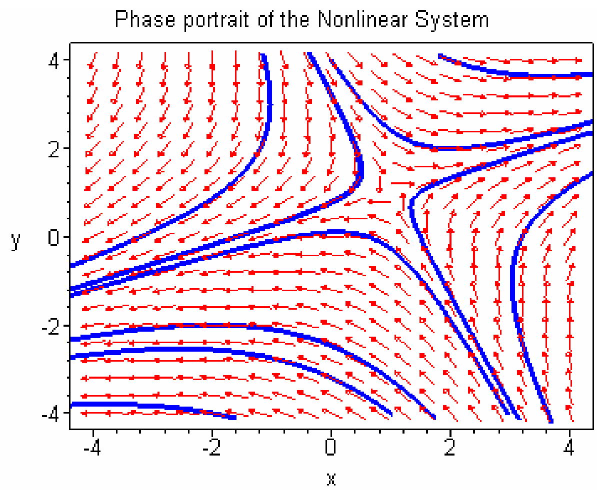
2.

1）解: $\begin{cases} x+y-2=0 \\ x-y=0 \end{cases}$ $\begin{cases} x=1 \\ y=1 \end{cases}$

奇点为(1, 1), 令 $X=x-1,\quad Y=y-1$代入 可判断奇点是鞍点

图中渐近线的斜率是: $\dfrac{dY}{dX}=K=\dfrac{1-K}{1+K},\quad K=-1\pm\sqrt{2}$

```
>restart:with(DEtools): a:=4:aa:=1: bb:=1: cc:=1: dd:=-1:
ODE1 :=[diff(x(t),t)=aa*x(t)+bb*y(t)-2,
diff(y(t),t)=cc*x(t)+dd*y(t)];
DEplot( ODE1, [x(t),y(t)],t=-20..20,
[[x(0)=0.1,y(0)=0.1],[x(0)=0,y(0)=0.8*a],
[x(0)=0,y(0)=-0.8*a],[x(0)=-a/2,y(0)=-a/2],
[x(0)=a/2,y(0)=a],[x(0)=-a/2,y(0)=-a],
[x(0)=a/2,y(0)=a/2],[x(0)=a/2,y(0)=a/3],
[x(0)=a/8,y(0)=a/3],
[x(0)=-0.8*a,y(0)=0],[x(0)=0.8*a,y(0)=0]],
 x=-a..a,y=-a..a,stepsize=0.05, dirgrid=[21,21],
 color=red, linecolor=blue,axes=BOXED,
 title="Phase portrait of the Nonlinear System",
arrows=SLIM);
```

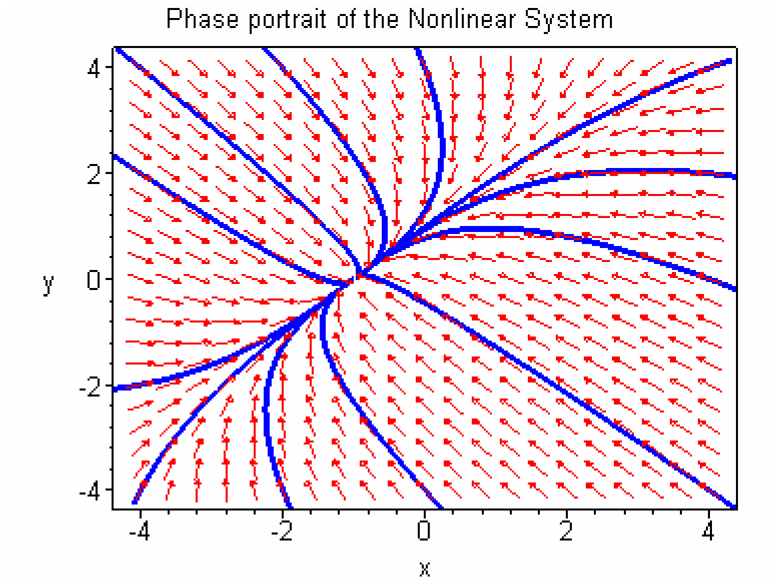


2）解：$\begin{cases} y-2x-2=0 \\ x-2y+1=0 \end{cases}$ $\begin{cases} x=-1 \\ y=0 \end{cases}$

令 $X=x+1,\quad Y=y$ 代入，可判断是稳定结点

$$K=\frac{1-2K}{-2+K},\quad K=\pm 1$$

```
ODE1 :=[diff(x(t),t)=aa*x(t)+bb*y(t)-2,
diff(y(t),t)=cc*x(t)+dd*y(t)+1];
DEplot( ODE1, [x(t),y(t)],t=-20..20,
[[x(0)=-a,y(0)=a],[x(0)=-a,y(0)=0.5*a],
[x(0)=0,y(0)=a],[x(0)=-a,y(0)=-a],
[x(0)=-a/2,y(0)=a],[x(0)=-a,y(0)=-a/2],
[x(0)=a,y(0)=a],[x(0)=-a/2,y(0)=-a],[x(0)=0,y(0)=-a],
[x(0)=a,y(0)=-a],[x(0)=a,y(0)=a/2],[x(0)=a,y(0)=0]],
 x=-a..a,y=-a..a,stepsize=0.05, dirgrid=[21,21],
 color=red, linecolor=blue,axes=BOXED,
 title="Phase portrait of the Linear System",
arrows=SLIM);
```



3）解：$\begin{cases} x+y+1=0 \\ 2x-y+5=0 \end{cases}$ $\begin{cases} x=-2 \\ y=1 \end{cases}$

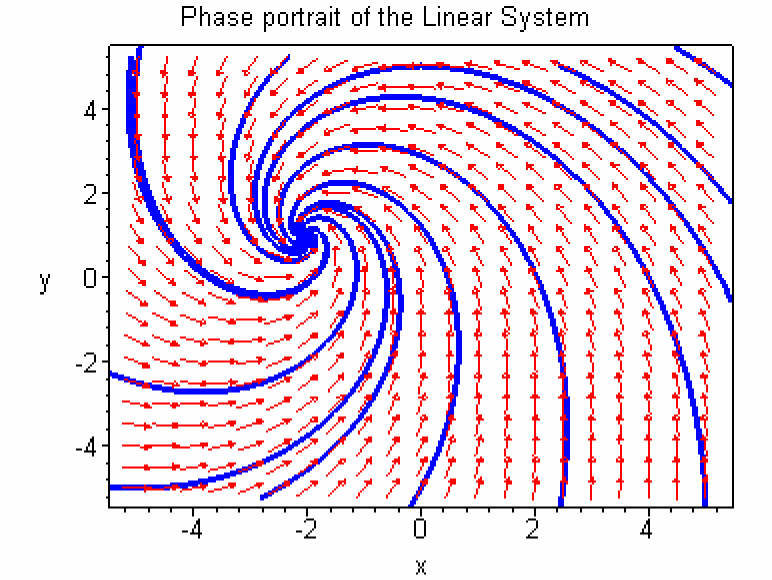
令 $X=x+2,\quad Y=y-1$，代入可判断是稳定焦点

取极坐标变换，$X=r\cos\theta,\quad Y=r\sin\theta$

$$\frac{dr}{dt}=r(\frac{1}{2}\sin 2\theta-1),\quad \frac{d\theta}{dt}=1+\cos^2\theta$$

```
diff(y(t),t)=cc*x(t)+dd*y(t)+5];
DEplot( ODE1, [x(t),y(t)],t=-20..20,
```

```
[[x(0)=-a,y(0)=a],[x(0)=-a,y(0)=0.5*a],
[x(0)=0,y(0)=a],[x(0)=-a,y(0)=-a],
[x(0)=-a/2,y(0)=a],[x(0)=-a,y(0)=-a/2],
[x(0)=a,y(0)=a],[x(0)=-a/2,y(0)=-a],[x(0)=0,y(0)=-a],
[x(0)=a,y(0)=-a],[x(0)=a,y(0)=a/2],[x(0)=a/2,y(0)=-a]],
 x=-a..a,y=-a..a,stepsize=0.05, dirgrid=[21,21],
 color=red, linecolor=blue,axes=BOXED,
 title="Phase portrait of the Linear System",
arrows=SLIM);
```



4）解: $\begin{cases}\alpha-\beta y=0\\ -\gamma+\delta x=0\end{cases}$ $\begin{cases}x=\dfrac{\gamma}{\delta}\\ y=\dfrac{\alpha}{\beta}\end{cases}$

令 $X=x-\dfrac{\gamma}{\delta}$, $Y=y-\dfrac{\alpha}{\beta}$ 判断是中心点，取极坐标变换，

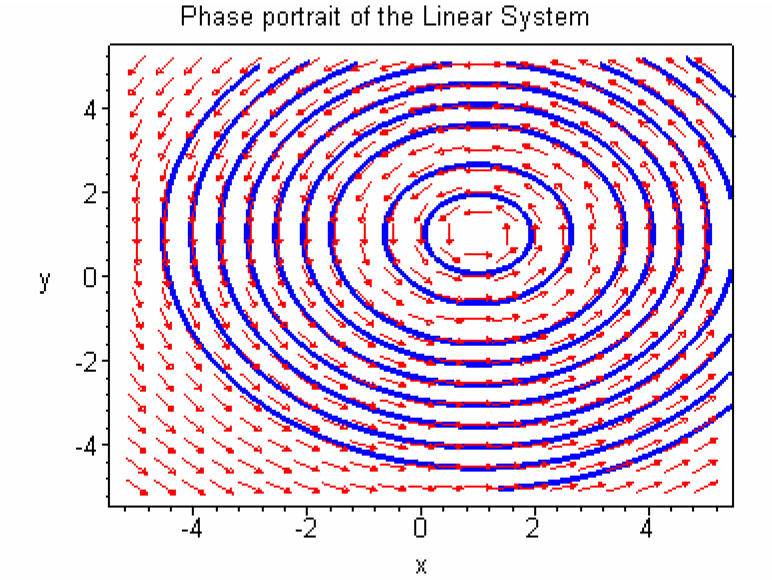
$X=r\cos\theta$, $Y=r\sin\theta$

$$\begin{cases}\dfrac{d\gamma}{dt}=\dfrac{1}{2}(\delta-\beta)\gamma\sin 2\theta\\ \dfrac{d\theta}{dt}=\beta\cos^2\theta+\delta\sin^2\theta>0\end{cases}$$

图形为逆时针方向旋转

```
> diff(y(t),t)=cc*x(t)+dd*y(t)-1];
DEplot( ODE1, [x(t),y(t)],t=-20..20,
[[x(0)=a/3,y(0)=a/3],[x(0)=a/3,y(0)=a/2],
[x(0)=a/3,y(0)=a],[x(0)=a/3,y(0)=-0.6*a],
[x(0)=a/3,y(0)=-0.7*a],[x(0)=a/3,y(0)=-0.8*a],
```

```
[x(0)=a/3,y(0)=-0.9*a],[x(0)=a/3,y(0)=-a],
[x(0)=a/3,y(0)=-1.2*a],[x(0)=a/3,y(0)=0.7*a],
x(0)=a/3,y(0)=0.8*a],[x(0)=a/3,y(0)=0.9*a]],
 x=-a..a,y=-a..a,stepsize=0.05, dirgrid=[21,21],
 color=red, linecolor=blue,axes=BOXED,
 title="Phase portrait of the Linear System",
arrows=SLIM);
```



3. 解：1）证明：$\begin{cases}\dfrac{dx}{dt}=ax+by\\ \dfrac{dy}{dt}=cx+dy\end{cases}$

$$\begin{vmatrix}a-\lambda & b\\ c & d-\lambda\end{vmatrix}=(a-\lambda)(d-\lambda)-bc=\lambda^2-(a+d)\lambda+ad-bc=\lambda[\lambda-a-d]$$

A 为常数矩阵 $|A|=\begin{vmatrix}a & b\\ c & d\end{vmatrix}=ad-bc=0$

∴ 系统没有孤立奇点，而非孤立奇点充满了一条直线.

2）由 1）知，$\lambda=0$，$\lambda=a+d$，解此方程组

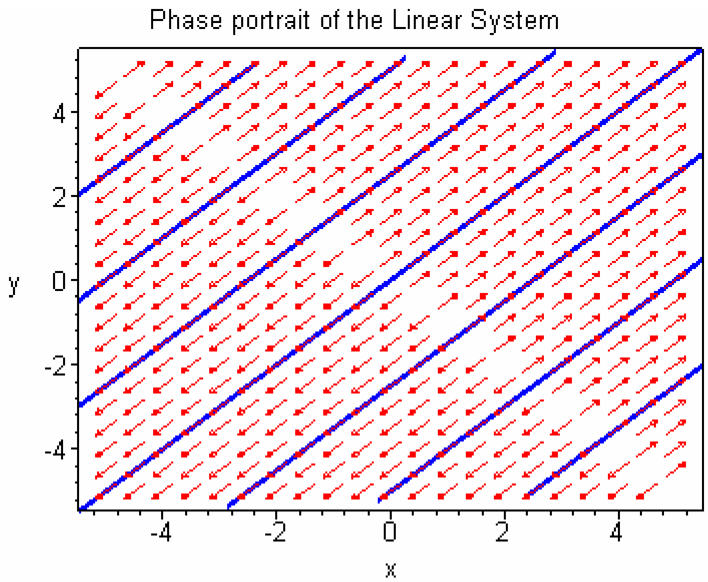
$$x=c_1b+c_2be^{(a+d)t},\qquad y=-c_1a+c_2de^{(a+d)t}$$

解轨线为：$$d(x-c_1b)=b(y+c_1a)$$

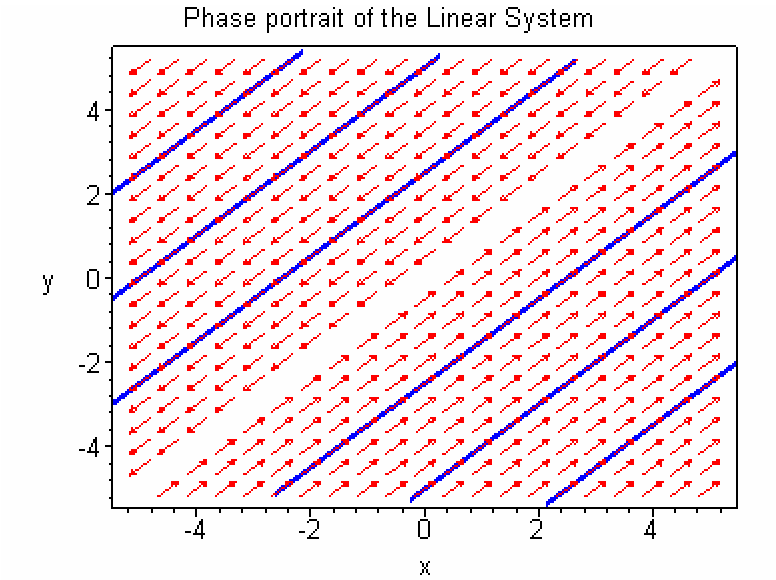
y 与 x 之间为线性关系，故相平面上轨线为一族平行线

```
 restart:with(DEtools): a:=5:aa:=1: bb:=1: cc:=1: dd:=1:
ODE1 :=[diff(x(t),t)=aa*x(t)+bb*y(t),
diff(y(t),t)=cc*x(t)+dd*y(t)];
```

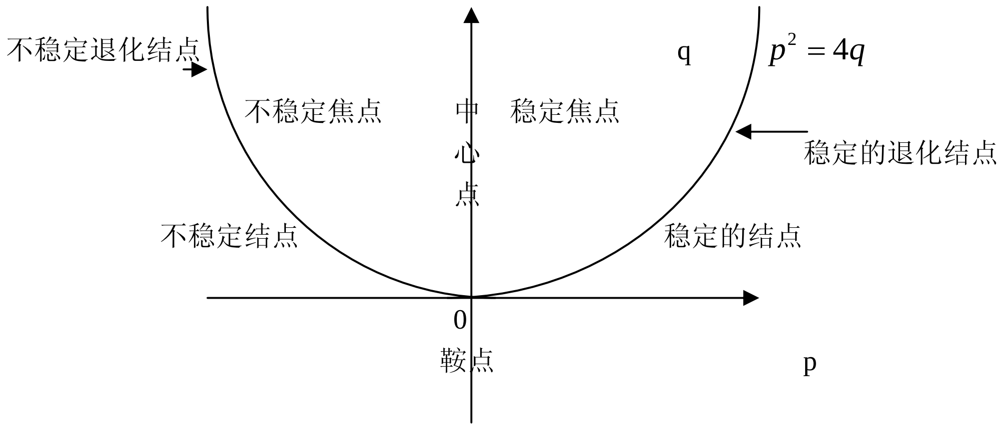
```
DEplot( ODE1, [x(t),y(t)],t=-20..20,
[[x(0)=a,y(0)=a],[x(0)=-a,y(0)=-a],
[x(0)=0,y(0)=a], [x(0)=a/2,y(0)=-a],
[x(0)=a/2,y(0)=a],[x(0)=-a/2,y(0)=a],
[x(0)=-a,y(0)=a/2],[x(0)=a,y(0)=a/2],
[x(0)=-a,y(0)=0],[x(0)=-a,y(0)=-a/2],
[x(0)=-a/2,y(0)=-a],[x(0)=0,y(0)=-a],
[x(0)=a,y(0)=-a/2],[x(0)=a,y(0)=0]],
 x=-a..a,y=-a..a,stepsize=0.05, dirgrid=[21,21],
 color=red, linecolor=blue,axes=BOXED,
 title="Phase portrait of the Linear System",
arrows=SLIM);
```




```
restart:with(DEtools): a:=5:aa:=1: bb:=-1: cc:=1: dd:=-1:
ODE1 :=[diff(x(t),t)=aa*x(t)+bb*y(t),
diff(y(t),t)=cc*x(t)+dd*y(t)];
DEplot( ODE1, [x(t),y(t)],t=-20..20,
[[x(0)=a,y(0)=a],[x(0)=-a,y(0)=-a],
[x(0)=0,y(0)=a], [x(0)=a/2,y(0)=-a],
[x(0)=a/2,y(0)=a],[x(0)=-a/2,y(0)=a],
[x(0)=-a,y(0)=a/2],[x(0)=a,y(0)=a/2],
[x(0)=-a,y(0)=0],[x(0)=-a,y(0)=-a/2],
[x(0)=-a/2,y(0)=-a],[x(0)=0,y(0)=-a],
[x(0)=a,y(0)=-a/2],[x(0)=a,y(0)=0]],
 x=-a..a,y=-a..a,stepsize=0.05, dirgrid=[21,21],
 color=red, linecolor=blue,axes=BOXED,
 title="Phase portrait of the Linear System",
arrows=SLIM);
```



4：



5. 解：原振于的振动方程可化为

$$\begin{cases} \dfrac{dx}{dt} = y \\ \dfrac{dy}{dt} = \dfrac{1}{m}(-kx - cy) \end{cases}$$

令 $\dfrac{dy}{dt} = \dfrac{dx}{dt} = 0$，可得（0，0）是奇点

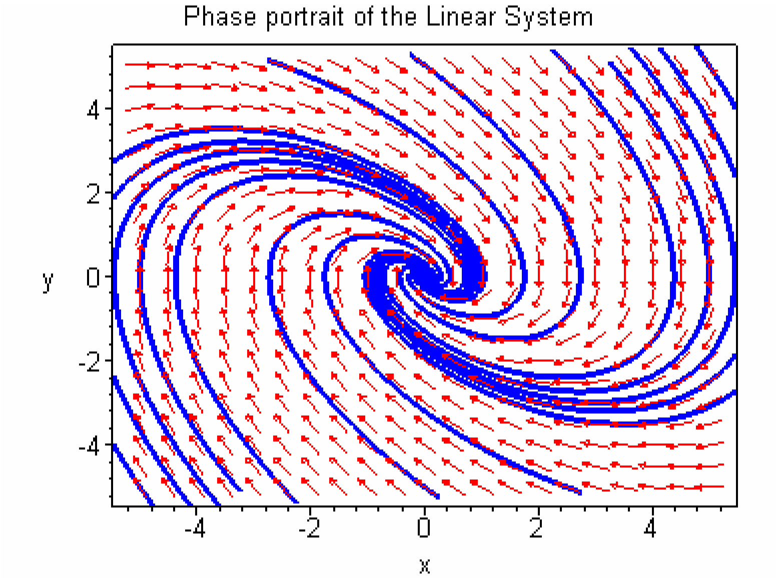
$p = \dfrac{c}{m} > 0$　　$q = \dfrac{k}{m} > 0$，　$\Delta = \dfrac{c^2}{m^2} - 4\dfrac{k}{m}$，

若 $\Delta = \dfrac{c^2}{m^2} - 4\dfrac{k}{m} > 0$，　奇点 $(0,0)$ 为稳定结点；

若 $\Delta = \dfrac{c^2}{m^2} - 4\dfrac{k}{m} < 0$，奇点（0，0）为稳定焦点

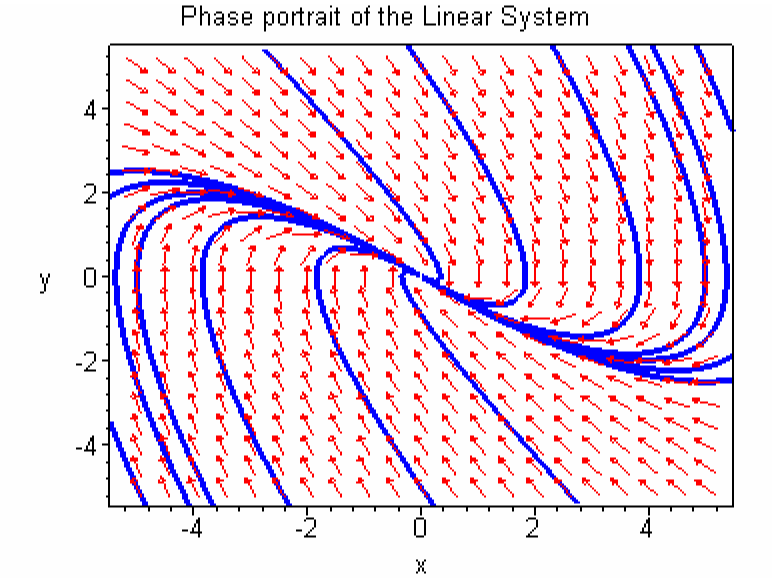
若$\Delta=\dfrac{c^2}{m^2}-4\dfrac{k}{m}=0$，奇点（0，0）为稳定临界—退化结点。

```
> restart:with(DEtools): a:=5: mk:=-1: mc:=-1:
ODE1 :=[diff(x(t),t)=y(t),
diff(y(t),t)=mk*x(t)+mc*y(t)];
DEplot( ODE1, [x(t),y(t)],t=-20..20,
[[x(0)=a,y(0)=a],[x(0)=-a,y(0)=-a],
[x(0)=0,y(0)=a], [x(0)=a/2,y(0)=-a],
[x(0)=a/2,y(0)=a],[x(0)=-a/2,y(0)=a],
[x(0)=-a,y(0)=a/2],[x(0)=a,y(0)=a/2],
[x(0)=-a,y(0)=0],[x(0)=-a,y(0)=-a/2],
[x(0)=-a/2,y(0)=-a],[x(0)=0,y(0)=-a],
[x(0)=a,y(0)=-a/2],[x(0)=a,y(0)=0]],
 x=-a..a,y=-a..a,stepsize=0.05, dirgrid=[21,21],
 color=red, linecolor=blue,axes=BOXED,
 title="Phase portrait of the Linear System",
arrows=SLIM);
```




```
> restart:with(DEtools): a:=5: mk:=-1: mc:=-2:
ODE1 :=[diff(x(t),t)=y(t),
diff(y(t),t)=mk*x(t)+mc*y(t)];
DEplot( ODE1, [x(t),y(t)],t=-20..20,
[[x(0)=a,y(0)=a],[x(0)=-a,y(0)=-a],
[x(0)=0,y(0)=a], [x(0)=a/2,y(0)=-a],
[x(0)=a/2,y(0)=a],[x(0)=-a/2,y(0)=a],
[x(0)=-a,y(0)=a/2],[x(0)=a,y(0)=a/2],
[x(0)=-a,y(0)=0],[x(0)=-a,y(0)=-a/2],
[x(0)=-a/2,y(0)=-a],[x(0)=0,y(0)=-a],
[x(0)=a,y(0)=-a/2],[x(0)=a,y(0)=0]],
 x=-a..a,y=-a..a,stepsize=0.05, dirgrid=[21,21],
```

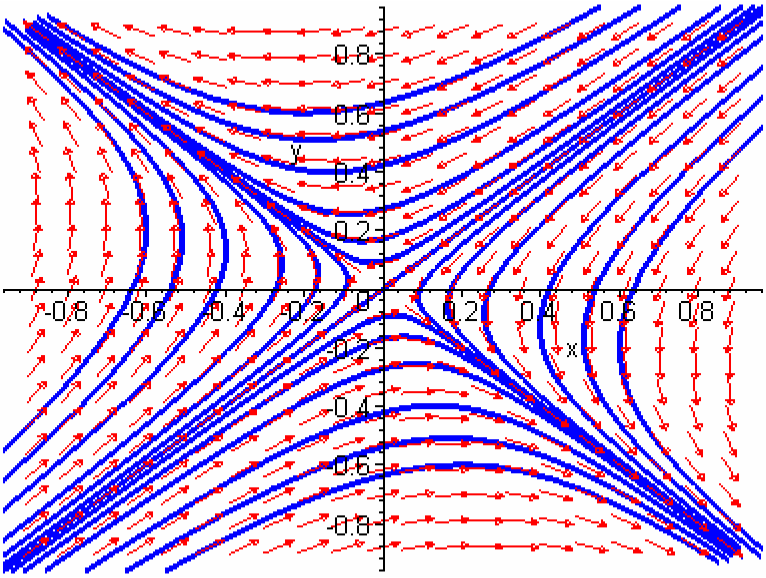
```
 color=red, linecolor=blue,axes=BOXED,
 title="Phase portrait of the Linear System",
arrows=SLIM);
```



6.

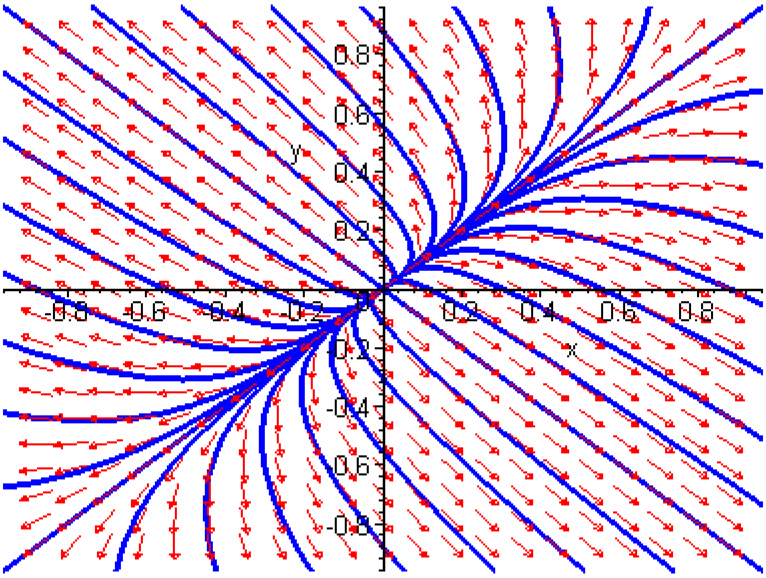
(1) c=-1

```
 with(DEtools):  a:=0.11; c:=-1;
DE931:=[diff(x(t),t)=c*x(t)-3*y(t),
diff(y(t),t)=-3*x(t)+c*y(t)]:
DEplot(DE931,[x(t),y(t)], t=-10..10,
[[x(0)=-8*a,y(0)=-8*a],[x(0)=-8*a,y(0)=-7.8*a],
[x(0)=-8*a,y(0)=-7.5*a],[x(0)=-8*a,y(0)=-7*a],
[x(0)=-8*a,y(0)=-6*a],[x(0)=-8*a,y(0)=-5*a],
[x(0)=-8*a,y(0)=-4*a],[x(0)=8*a,y(0)=8*a],
[x(0)=8*a,y(0)=7.8*a],[x(0)=8*a,y(0)=7.5*a],
[x(0)=8*a,y(0)=7*a],[x(0)=8*a,y(0)=6*a],
[x(0)=8*a,y(0)=5*a],[x(0)=8*a,y(0)=4*a],
[x(0)=4*a,y(0)=8*a],[x(0)=5*a,y(0)=8*a],
[x(0)=6*a,y(0)=8*a],[x(0)=7*a,y(0)=8*a],
[x(0)=7.5*a,y(0)=8*a],[x(0)=7.8*a,y(0)=8*a],
[x(0)=8*a,y(0)=8*a],[x(0)=-7.8*a,y(0)=-8*a],
[x(0)=-7.5*a,y(0)=-8*a],[x(0)=-7*a,y(0)=-8*a],
[x(0)=-6*a,y(0)=-8*a],[x(0)=-5*a,y(0)=-8*a],
[x(0)=-4*a,y(0)=-8*a]],
x=-8*a..8*a,y=-8*a..8*a, stepsize=0.05,
dirgrid=[21,21], color=red,linecolor=blue,
arrows=SLIM);
```



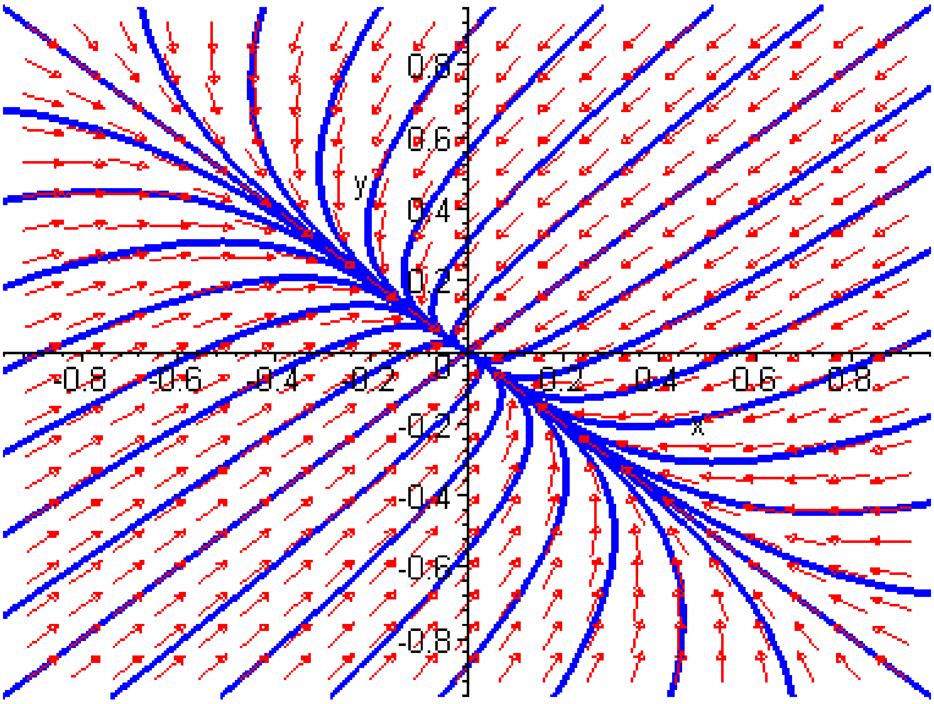
(2) c=5

```
with(DEtools):  a:=0.11;
c:=5; DE931:=[diff(x(t),t)=c*x(t)-3*y(t),
diff(y(t),t)=-3*x(t)+c*y(t)]:
DEplot(DE931,[x(t),y(t)], t=-10..10,
[[x(0)=-8*a,y(0)=-8*a],[x(0)=-8*a,y(0)=-6*a],
[x(0)=-8*a,y(0)=-4*a],[x(0)=-8*a,y(0)=-2*a],
[x(0)=-8*a,y(0)=0],[x(0)=-8*a,y(0)=2*a],
[x(0)=-8*a,y(0)=4*a],[x(0)=-8*a,y(0)=6*a],
[x(0)=-8*a,y(0)=8*a],[x(0)=8*a,y(0)=-8*a],
[x(0)=8*a,y(0)=-6*a],[x(0)=8*a,y(0)=-4*a],
[x(0)=8*a,y(0)=-2*a],[x(0)=8*a,y(0)=0],
[x(0)=8*a,y(0)=2*a],[x(0)=8*a,y(0)=4*a],
[x(0)=8*a,y(0)=6*a],[x(0)=8*a,y(0)=8*a],
[x(0)=-6*a,y(0)=-8*a],[x(0)=-4*a,y(0)=-8*a],
[x(0)=-2*a,y(0)=-8*a],[x(0)=0,y(0)=-8*a],
[x(0)=2*a,y(0)=-8*a],[x(0)=4*a,y(0)=-8*a],
[x(0)=6*a,y(0)=-8*a],[x(0)=-8*a,y(0)=-8*a],
[x(0)=-6*a,y(0)=8*a],[x(0)=-4*a,y(0)=8*a],
[x(0)=-2*a,y(0)=8*a],[x(0)=0,y(0)=8*a],
[x(0)=2*a,y(0)=8*a],[x(0)=4*a,y(0)=8*a],
[x(0)=6*a,y(0)=8*a]],
x=-8*a..8*a,y=-8*a..8*a, stepsize=0.05,
dirgrid=[21,21], color=red,linecolor=blue,
arrows=SLIM);
```



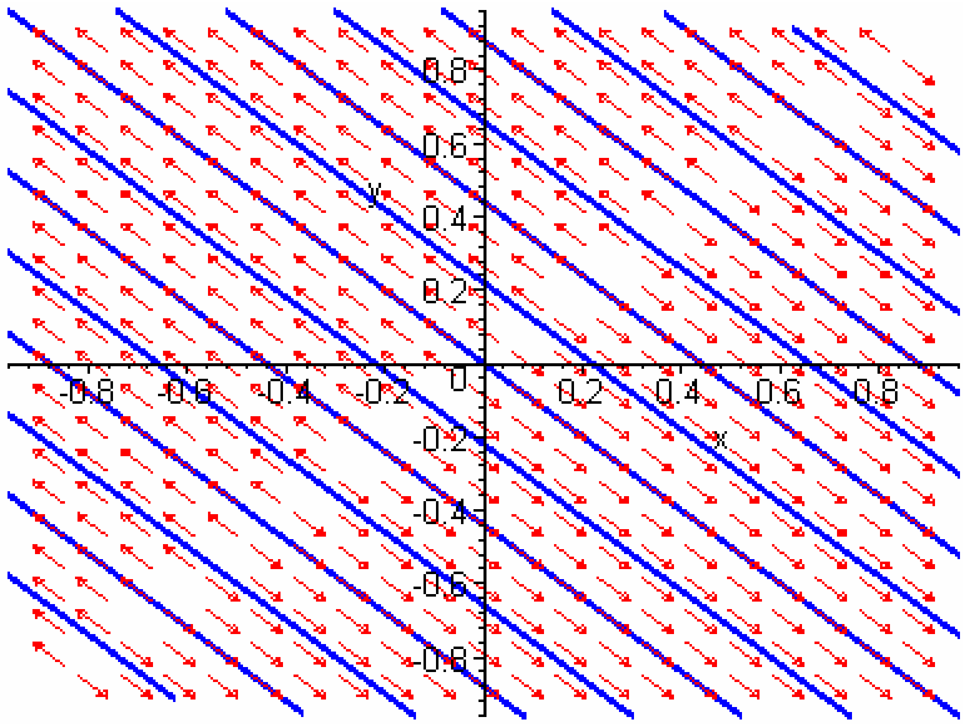
(3) c=-5

```
> with(DEtools):  a:=0.11;
c:=-5; DE931:=[diff(x(t),t)=c*x(t)-3*y(t),
diff(y(t),t)=-3*x(t)+c*y(t)]:
DEplot(DE931,[x(t),y(t)], t=-10..10,
[[x(0)=-8*a,y(0)=-8*a],[x(0)=-8*a,y(0)=-6*a],
[x(0)=-8*a,y(0)=-4*a],[x(0)=-8*a,y(0)=-2*a],
[x(0)=-8*a,y(0)=0],[x(0)=-8*a,y(0)=2*a],
[x(0)=-8*a,y(0)=4*a],[x(0)=-8*a,y(0)=6*a],
[x(0)=-8*a,y(0)=8*a],[x(0)=8*a,y(0)=-8*a],
[x(0)=8*a,y(0)=-6*a],[x(0)=8*a,y(0)=-4*a],
[x(0)=8*a,y(0)=-2*a],[x(0)=8*a,y(0)=0],
[x(0)=8*a,y(0)=2*a],[x(0)=8*a,y(0)=4*a],
[x(0)=8*a,y(0)=6*a],[x(0)=8*a,y(0)=8*a],
[x(0)=-6*a,y(0)=-8*a],[x(0)=-4*a,y(0)=-8*a],
[x(0)=-2*a,y(0)=-8*a],[x(0)=0,y(0)=-8*a],
[x(0)=2*a,y(0)=-8*a],[x(0)=4*a,y(0)=-8*a],
[x(0)=6*a,y(0)=-8*a],[x(0)=-8*a,y(0)=-8*a],
[x(0)=-6*a,y(0)=8*a],[x(0)=-4*a,y(0)=8*a],
[x(0)=-2*a,y(0)=8*a],[x(0)=0,y(0)=8*a],
[x(0)=2*a,y(0)=8*a],[x(0)=4*a,y(0)=8*a],
[x(0)=6*a,y(0)=8*a]],
x=-8*a..8*a,y=-8*a..8*a, stepsize=0.05,
dirgrid=[21,21], color=red,linecolor=blue,
arrows=SLIM);
```



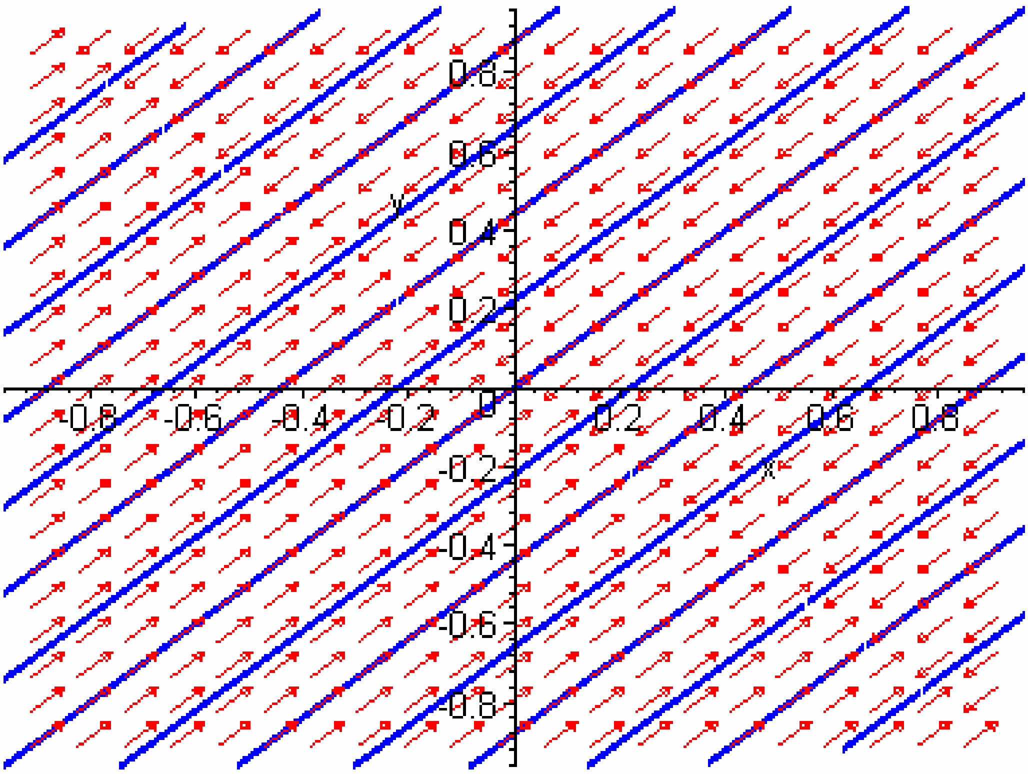
y
0.8
0.6
0.4
0.2
0
-0.8
-0.6
-0.4
-0.2
0.2
0.4
0.6
0.8
x
-0.2
-0.4
-0.6
-0.8

（4） c=3



0.8
0.6
y
0.4
0.2
0
-0.8
-0.6
-0.4
-0.2
0.2
0.4
0.6
0.8
x
-0.2
-0.4
-0.6
-0.8

(5) c=-3



0.8
0.6
y
0.4
0.2
0
-0.8
-0.6
-0.4
-0.2
0.2
0.4
0.6
0.8
-0.2
x
-0.4
-0.6
-0.8