习题2.1

1. 1) $y = ce^{-\frac{(\ln x)^2}{2}}$ ，

2) $y = ce^{\frac{1}{x}}$,

3) $y = \frac{-1}{2}(\sin x + \cos x) + ce^x$

4) $y = \sin x - 1 + ce^{-\sin x}$,

5) $y = \dfrac{x^2}{e^x(-x^2 + 2x - 2) + C}$

6) $y = x(3x + c)^{1/3}$ ，

7) $y = (x+1)^2(x^2/2 + x + c)$

8) $2x = cy + y^3$ 及 $y=0$,

9) $y = cx^a + \dfrac{x}{1-a} - \dfrac{1}{a}, (a \neq 0,1)$；$x + \ln|x| + c, (a = 0)$；$cx + x\ln|x| - 1, a = 1$

10) $y^2(x^2 + 1 + ce^{x^2}) = 1$ 及 $y = 0$,

11) $y(cx^2 + 1 + 2\ln x) = 4, y = 0$

12) $y^2 = x + cx^2$,

13，$\dfrac{x^2}{2} + x^3 e^{-y} = c$

14 $(1 - x^2 + x^2 y^2)e^{y^2} = cx^2$,

15) $y = (1 + x)e^x$

2．1) $y = 2(\sec(kx))^{-1/k}$,

2) $y = \dfrac{e}{x\ln x}$,

3) $y = \dfrac{2x^4 + 1}{x^2}$

4）$y = \dfrac{\ln(x-1) - \cos x + 1 + \cos(2)}{x^3 - 3x^2 + 3x - 1}$

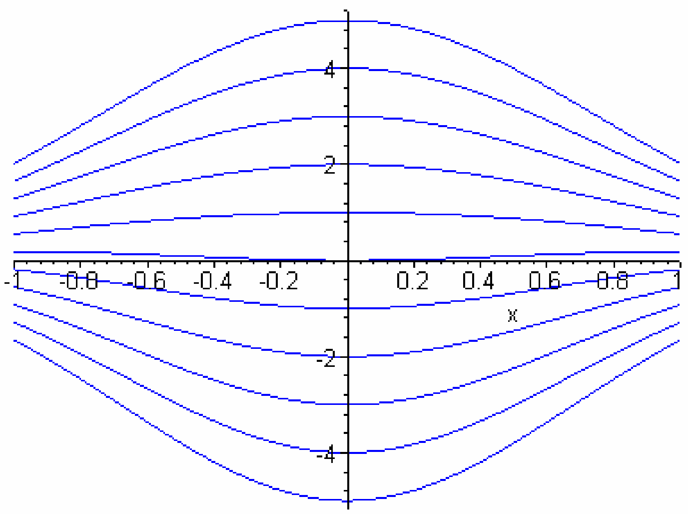
3.程序如下，图略

1）

```
> restart:dsolve(diff(y(x),x)+2*x*y(x)-x*exp(-x^2));
  assign(%);
  sols:={seq(subs(_C1=j,y(x)),j=-5..5)};
  plot(sols,x=-1..1,color=blue);
```

$$y(x) = \left(\frac{1}{2}x^2 + \_C1\right) e^{(-x^2)}$$

$$sols := \left\{\frac{1}{2} e^{(-x^2)} x^2, \left(\frac{1}{2}x^2 + 4\right) e^{(-x^2)}, \left(\frac{1}{2}x^2 + 5\right) e^{(-x^2)}, \left(\frac{1}{2}x^2 - 5\right) e^{(-x^2)}, \left(\frac{1}{2}x^2 - 4\right) e^{(-x^2)}, \left(\frac{1}{2}x^2 - 3\right) e^{(-x^2)}, \left(\frac{1}{2}x^2 - 2\right) e^{(-x^2)}, \left(\frac{1}{2}x^2 - 1\right) e^{(-x^2)}, \left(\frac{1}{2}x^2 + 1\right) e^{(-x^2)}, \left(\frac{1}{2}x^2 + 2\right) e^{(-x^2)}, \left(\frac{1}{2}x^2 + 3\right) e^{(-x^2)}\right\}$$

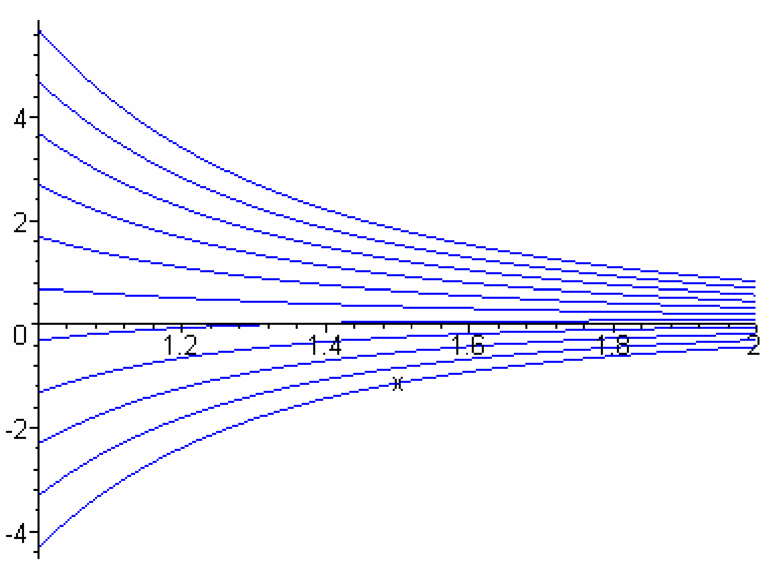


（2）

```
> restart:dsolve(x*diff(y(x),x)+3*y(x)=2/x/(1+x^2));
  assign(%);
  sols:={seq(subs(_C1=j,y(x)),j=-5..5)};
  plot(sols,x=1..2,color=blue);
```

$$y(x) = \frac{\ln(1+x^2) + \_C1}{x^3}$$

$$sols := \left\{\frac{\ln(1+x^2)}{x^3}, \frac{\ln(1+x^2)-5}{x^3}, \frac{\ln(1+x^2)-4}{x^3}, \frac{\ln(1+x^2)-3}{x^3}, \frac{\ln(1+x^2)-2}{x^3}, \frac{\ln(1+x^2)-1}{x^3}, \frac{\ln(1+x^2)+1}{x^3}, \frac{\ln(1+x^2)+2}{x^3}, \frac{\ln(1+x^2)+3}{x^3}, \frac{\ln(1+x^2)+4}{x^3}, \frac{\ln(1+x^2)+5}{x^3}\right\}$$



4.

设人体吸收葡萄糖得速率与血中葡萄糖得含量成比例得比例系数为 k，设以常数 c 速率注射，设任意时刻得葡萄糖得含量为 G(t),则 Δt 时间内

$G(t+\Delta t)-G(t)=-kG(t)\Delta t+c\Delta t$，则得到微分方程：

$\dfrac{dG(t)}{dt}=-kG(t)+c,\qquad G(0)=G_0$，解此微分方程得到：$G(t)=\dfrac{c}{k}+e^{-kt}\left(G_0-\dfrac{c}{k}\right)$

5.

设任意时刻车间内的CO_2的含量为x(t),

$$x(t+\Delta t)-x(t)=0.005\Delta t-\frac{x(t)\Delta t}{900},\quad x(0)=18$$

得到微分方程：$\dfrac{dx(t)}{dt}=0.005-\dfrac{x}{900},\qquad x(0)=18,$

解此微分方程：$x(t)=\dfrac{9}{2}+\dfrac{27}{2}e^{-t/900}$

t时刻车间内的CO_2的百分比为 $\dfrac{x(t)}{9000}=\dfrac{1}{2000}+\dfrac{3}{2000}e^{-t/900}$

则 20 分钟后车间内的CO_2的百分比为：

$$\frac{x(20)}{9000}=\frac{1}{2000}+\frac{3}{2000}e^{-20/900}=0.001967$$

6. $y(x)=\left(\displaystyle\int_{x0}^{x} f(u)\,\mathbf{e}^{(k\,u)}\,du+\frac{y0}{\mathbf{e}^{(-k\,x0)}}\right)\mathbf{e}^{(-k\,x)}$ $x\to\infty$时，$y\to L/k$

7.显然$y_1'+p(x)y_1=g_1(x)$, $y_2'+p(x)y_2=g_2(x)$把$y=y_1+y_2$代入得,

左边$=y'+p(x)y=y_1'+y_2'+p(x)y_1+p(x)y_2=g_1(x)+g_2(x)=$右，

则y_1+y_2是方程$y'+p(x)y=g_1(x)+g_2(x)$的解

8.证明：（1）方程的解是 $y=y_0\exp\left(-\int_{x_0}^{x}p(s)ds\right)$，由题意，如果 y 是周期解，

即 $y(x)=y(x+\omega)$，则带入就可以得到 $\int_0^{\omega}p(x)dx=0$

（2）方程的通解是 $y=\exp\left(-\int_{x_0}^{x}p(s)ds\right)\left(\int_{x_0}^{x}g(s)\exp\left(\int_{x_0}^{s}p(\tau)d\tau\right)ds+c\right)$，

在题目给定的条件下，周期解应该满足 $y(x)=y(x+T)$，特别 $y(x_0)=y(x+T_0)$，代入到通解中，得到 $c=\dfrac{\exp\left(-\int_{x_0}^{x_0+\omega}p(\tau)d\tau\right)\int_{x_0}^{x_0+\omega}g(s)\exp\left(\int_{x_0}^{s}p(\tau)d\tau\right)ds}{1-\exp\left(-\int_{x_0}^{x_0+\omega}p(\tau)d\tau\right)}$，代入回通解中验证即得到结论。

9.证明：通解是 $y(x)=e^{-x}\left(\int_{x_0}^{x}f(s)e^{s}ds+c\right)=\dfrac{\int_{x_0}^{x}f(s)e^{s}ds+c}{e^{x}}$，对于有界解，

由于分母当 $x\to-\infty$时，$e^{x}\to 0$，故分子必然趋于 0，即 $c=\int_{-\infty}^{x_0}f(s)e^{s}ds$，

所以这个惟一的有界解为 $y(x)=\dfrac{\int_{-\infty}^{x}f(s)e^{s}ds}{e^{x}}$，其有界性可以通过对函数 f(x)放大的方法得到。周期性验证与上面一题相同.

10.根据题目的提示：

1） $\tan y=ce^{3x}-1/3$

2） $e^{y^2}=\dfrac{c+x}{x^2}$，

3） $y=e^{(x^2+\frac{c}{x^2})}$，

4） $y=\dfrac{2x}{cx^2-3}-1$