答 案 4.5

1. (1) $x(t)=c_1\begin{pmatrix}1\\-1\end{pmatrix}e^{2t}+c_2\begin{pmatrix}4\\1\end{pmatrix}e^{-3t}+\begin{pmatrix}t^2+t\\-\frac{1}{2}t^2\end{pmatrix}$

(2) $x(t)=c_1\begin{pmatrix}1\\2\end{pmatrix}e^{2t}+c_2\begin{pmatrix}1\\0\end{pmatrix}e^{3t}+\begin{pmatrix}t^2+t-\frac{1}{6}\\t\end{pmatrix}$

(3) $x(t)=c_1\begin{pmatrix}\cos 2t\\\sin 2t\end{pmatrix}+c_2\begin{pmatrix}\sin 2t\\-\cos 2t\end{pmatrix}+\begin{pmatrix}t\\1\end{pmatrix}$

(4) $x(t)=c_1\begin{pmatrix}1\\1\end{pmatrix}e^{t}+c_2\begin{pmatrix}1\\-1\end{pmatrix}e^{-t}+\frac{1}{2}\begin{pmatrix}t-\frac{1}{2}\\t+\frac{1}{2}\end{pmatrix}e^{t}+\frac{1}{2}e^{-t}\begin{pmatrix}t+\frac{1}{2}\\-t+\frac{1}{2}\end{pmatrix}$

(5) $x(t)=c_1\begin{pmatrix}1\\-1\end{pmatrix}e^{-4t}+c_2\begin{pmatrix}t\\-t-1\end{pmatrix}e^{-4t}+\frac{e^{t}}{25}\begin{pmatrix}4\\1\end{pmatrix}+\frac{e^{2t}}{36}\begin{pmatrix}-1\\7\end{pmatrix}$

(6) $x(t)=c_1\begin{pmatrix}1\\-1\\0\end{pmatrix}e^{t}+c_2\begin{pmatrix}\sin t\\\cos t\\\sin t\end{pmatrix}+c_3\begin{pmatrix}\cos t\\-\sin t\\\cos t\end{pmatrix}+\begin{pmatrix}0\\t\\1\end{pmatrix}$

(7) $x(t)=c_1\begin{pmatrix}1\\1\\-1\end{pmatrix}e^{-t}+c_2\begin{pmatrix}1\\1\\2\end{pmatrix}e^{2t}+c_3\begin{pmatrix}1\\-1\\0\end{pmatrix}e^{-2t}+\begin{pmatrix}\frac{1}{6}e^{t}+\frac{3}{20}e^{3t}-2\\-\frac{1}{6}e^{t}+\frac{7}{20}e^{3t}-2\\-\frac{1}{2}e^{t}+\frac{1}{4}e^{3t}\end{pmatrix}$

(8) $x(t)=c_1\begin{pmatrix}t^2\\-2t\\2\end{pmatrix}e^{-t}+c_2\begin{pmatrix}t\\-1\\0\end{pmatrix}e^{-t}+c_3\begin{pmatrix}1\\0\\0\end{pmatrix}e^{-t}+\begin{pmatrix}t^2-3t+3\\t\\t-1\end{pmatrix}$

2. (1) $x=c_1\begin{pmatrix}3\\1\end{pmatrix}e^{4t}+c_2\begin{pmatrix}1\\1\end{pmatrix}e^{2t}+\begin{pmatrix}-4e^{3t}-e^{-t}\\-2e^{3t}-2e^{-t}\end{pmatrix}$

(2) $x=c_1\begin{pmatrix}4\\1\end{pmatrix}e^{t}+c_2\begin{pmatrix}1\\1\end{pmatrix}e^{-2t}+\begin{pmatrix}-4te^{t}\\(1-t)e^{t}\end{pmatrix}$

(3) $x=c_1\begin{pmatrix}2\\1\end{pmatrix}e^{2t}+c_2\begin{pmatrix}1\\-2\end{pmatrix}e^{-3t}+\begin{pmatrix}-(12t+13)\\-(8t+6)\end{pmatrix}e^{t}$

(4) $x=c_1\begin{pmatrix}1\\-1\end{pmatrix}e^{-t}+c_2\begin{pmatrix}1\\1\end{pmatrix}e^{5t}+\begin{pmatrix}2t-\frac{13}{5}\\-3t+\frac{12}{5}\end{pmatrix}+\begin{pmatrix}-3\\1\end{pmatrix}e^{t}$

(5) $x=c_1\begin{pmatrix}1\\1\end{pmatrix}e^{2t}+c_2\begin{pmatrix}1\\-3\end{pmatrix}e^{-2t}+\begin{pmatrix}-\frac{1}{4}-\frac{2}{3}e^{t}\\-\frac{3}{4}-e^{t}\end{pmatrix}$

(6) $x=c_1\begin{pmatrix}3\\1\\2\end{pmatrix}+c_2\begin{pmatrix}4\sin t+9\cos t\\\sin t+3\cos t\\2\sin t+7\cos t\end{pmatrix}+c_3\begin{pmatrix}9\sin t-4\cos t\\3\sin t-\cos t\\7\sin t-2\cos t\end{pmatrix}+\begin{pmatrix}-3t\\-t\\-2t\end{pmatrix}$

(7) $x=c_1\begin{pmatrix}1\\-1\\0\end{pmatrix}e^{2t}+c_2\begin{pmatrix}-2\\-3\\2\end{pmatrix}e^{-t}+c_3\begin{pmatrix}-2\\0\\1\end{pmatrix}e^{-3t}+\begin{pmatrix}-2t\\-3t\\2t\end{pmatrix}e^{-t}$

3. (1) $\begin{cases}x=2c_1e^{-2t}+c_2e^{3t}+1\\y=c_1e^{-2t}+3c_2e^{3t}+1\end{cases}$

(2) $\begin{cases}x=-\frac{1}{15}e^{-2t}+\frac{13}{12}e^{-t}+\frac{1}{6}e^{t}+\frac{2}{3}e^{2t}+\frac{3}{20}e^{3t}-2\\y=\frac{1}{15}e^{-2t}+\frac{13}{12}e^{-t}+\frac{2}{3}e^{2t}+\frac{7}{20}e^{3t}-\frac{1}{6}e^{t}-2\\z=-\frac{13}{12}e^{-t}+\frac{4}{3}e^{2t}-\frac{1}{2}e^{t}+\frac{1}{4}e^{3t}\end{cases}$

4. 证明：$\frac{dy}{dx}=\frac{dy}{dt}\frac{dt}{dx}$ $\quad\frac{dy}{dx}=\frac{dy}{dt}e^{-t}=e^{-t}(Ay+Q(e^{t}))$

所以原方程组化为$\frac{dy}{dt}=Ay+Q(e^{t})$。

5. 解：令$t=e^{s}$，则原方程组化为$\frac{\mathrm{d}x}{\mathrm{d}s}=Ax+\begin{pmatrix}e^{s}\\0\\1\end{pmatrix}$，再求解。

$$\begin{cases}x_1(t)=\frac{-c_1}{t}+c_2t^2-\frac{c_3}{t}-\frac{1}{2}\\x_2(t)=\frac{c_1}{t}+c_2t^2-\frac{1}{2}-\frac{1}{2}t\\x_3(t)=c_2t^2+\frac{c_3}{t}+\frac{1}{2}-\frac{1}{2}t\end{cases}$$

6.

（1）

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restart:
eq1:=diff(x(t),t)=x(t)/2-y(t)+t^2;
eq2:=diff(y(t),t)=x(t)+3*y(t)-t;
dsolve({eq1,eq2},{x(t),y(t)});
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$$eq1 := \frac{\partial}{\partial t}\mathrm{x}(t) = \frac{1}{2}\mathrm{x}(t) - \mathrm{y}(t) + t^2$$

$$eq2 := \frac{\partial}{\partial t}\mathrm{y}(t) = \mathrm{x}(t) + 3\,\mathrm{y}(t) - t$$

$$\{\mathrm{x}(t) = \mathbf{e}^{t}\,\_C2 + \mathbf{e}^{(5/2\,t)}\,\_C1 - \frac{258}{125} - \frac{54}{25}t - \frac{6}{5}t^2,$$

$$\mathrm{y}(t) = -\frac{1}{2}\mathbf{e}^{t}\,\_C2 - 2\,\mathbf{e}^{(5/2\,t)}\,\_C1 + \frac{141}{125} + \frac{33}{25}t + \frac{2}{5}t^2\}$$

（2）

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restart:
eq1:=diff(x(t),t)=-2*x(t)+exp(-2*t)*cos(t);
eq2:=diff(y(t),t)=5*x(t)+3*y(t);
dsolve({eq1,eq2},{x(t),y(t)});
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$$eq1 := \frac{\partial}{\partial t}\mathrm{x}(t) = -2\,\mathrm{x}(t) + \mathbf{e}^{(-2\,t)}\cos(t)$$

$$eq2 := \frac{\partial}{\partial t}\mathrm{y}(t) = 5\,\mathrm{x}(t) + 3\,\mathrm{y}(t)$$

$$\{\mathrm{x}(t) = \mathbf{e}^{(-2\,t)}(\sin(t) + \_C2),$$

$$\mathrm{y}(t) = -\frac{5}{26}\mathbf{e}^{(-2\,t)}\cos(t) - \frac{25}{26}\mathbf{e}^{(-2\,t)}\sin(t) - \mathbf{e}^{(-2\,t)}\,\_C2 + \mathbf{e}^{(3\,t)}\,\_C1\}$$

（3）

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> restart:
eq1:=diff(x(t),t)=x(t)-2*y(t)+10*cos(t);
eq2:=diff(y(t),t)=3*x(t)-4*y(t)-5*sin(t);
dsolve({eq1,eq2},{x(t),y(t)});
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$$eq1 := \frac{\partial}{\partial t}\mathrm{x}(t) = \mathrm{x}(t) - 2\,\mathrm{y}(t) + 10\cos(t)$$

$$eq2 := \frac{\partial}{\partial t}\mathrm{y}(t) = 3\,\mathrm{x}(t) - 4\,\mathrm{y}(t) - 5\sin(t)$$

$$\{\mathrm{x}(t) = 4\cos(t) + 12\sin(t) - \mathbf{e}^{(-2\,t)}\,\_C1 + \mathbf{e}^{(-t)}\,\_C2,$$

$$\mathrm{y}(t) = 8\sin(t) + \cos(t) - \frac{3}{2}\mathbf{e}^{(-2\,t)}\,\_C1 + \mathbf{e}^{(-t)}\,\_C2\}$$