答案 2.7

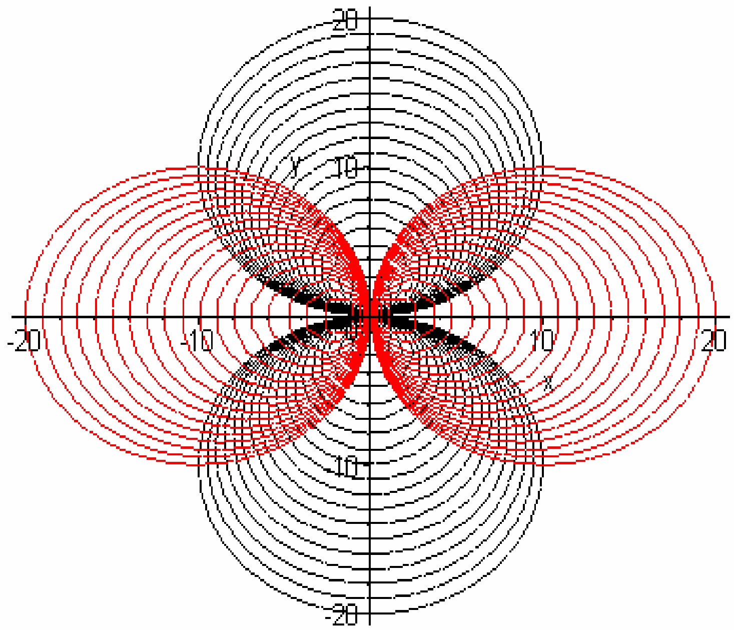
1.

（1）原曲线的斜率为 $y'=\dfrac{y^2-x^2}{2xy}$

所求得正交轨线满足微分方程：$y'=-\dfrac{2xy}{y^2-x^2}$

解该微分方程：$x^2+y^2=cy$

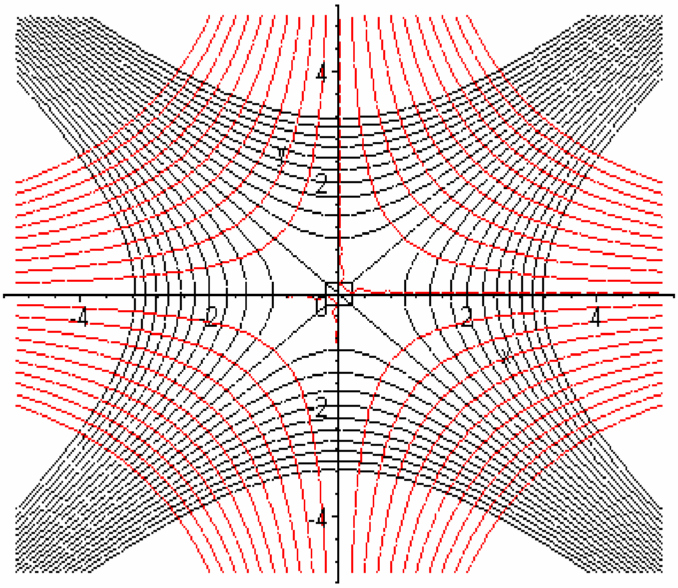
```
> restart:
with(plots):
plot1:=seq(implicitplot(x^2+y^2=c*x,
x=-20..20,y=-20..20,color=red),c=-20..20):
plot2:=seq(implicitplot(x^2+y^2=c*y,
x=-20..20,y=-20..20,color=black),c=-20..20):
display(plot1,plot2);
```



（2） dy/dx=-y/x，所求得正交轨线满足微分方程：dy/dx=x/y,

解该微分方程：$y^2-x^2+c=0$

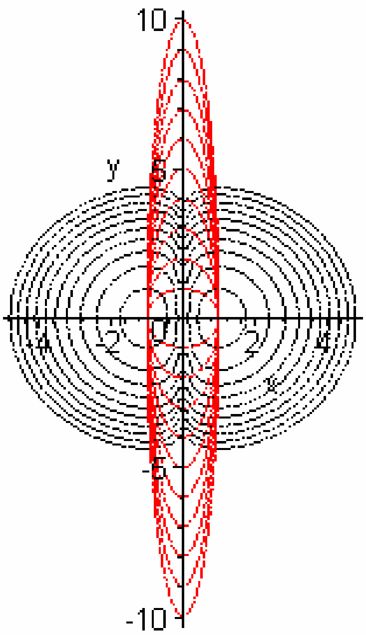
```
restart:
with(plots): a:=10;
plot1:=seq(implicitplot(x*y=c,
x=-a/2..a/2,y=-a/2..a/2,color=red),c=-a..a):
plot2:=seq(implicitplot(y^2-x^2=c,
x=-a/2..a/2,y=-a/2..a/2,color=black),c=-a..a):
display(plot1,plot2);
```



(3) dy/dx=xy/(x^2-1),所求得正交轨线满足微分方程：dy/dx=(1-x^2)/(xy),

解该微分方程： $y^2 - \ln x^2 + x^2 + c = 0$

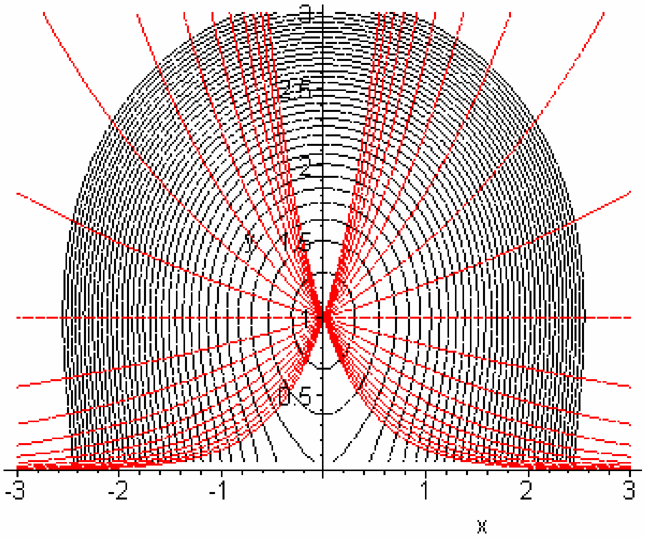
```
restart:
with(plots): a:=10:
plot1:=seq(implicitplot(x^2+y^2/c^2=1,
x=-a..a,y=-a..a,color=red,grid=[100,100]),c=1..10):
plot2:=seq(implicitplot(x^2+y^2-ln(x^2)=2*c,
x=-a..a,y=-a..a,color=black,grid=[100,100]),c=-10..10):
display(plot1,plot2);
```



(4) dy/dx=ylny/x,所求得正交轨线满足微分方程：dy/dx=-x/(ylny),

解该微分方程：$\frac{1}{2}x^2+\frac{1}{2}y^2\ln y-\frac{1}{4}y^2+c=0$

```
> restart:
with(plots): a:=3:
plot1:=seq(implicitplot(y=exp(c*x/5),
x=-a..a,y=-a..a,color=red,grid=[50,50]),c=-10..10):
plot2:=seq(implicitplot(x^2+y^2*ln(y)-y^2/2=c/5,
x=-a..a,y=-a..a,color=black,grid=[50,50]),c=-10..30):
display(plot1,plot2);
```

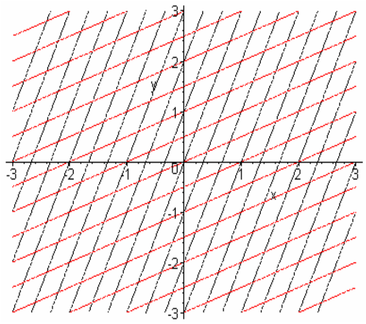


2. dy/dx=(H(x,y)+tanα)/(1-H(x,y)tanα)

1) 满足与已知曲线族相交成45度角的曲线族满足如下的微分方程：

dy/dx=3,解得：$y=3x+c$

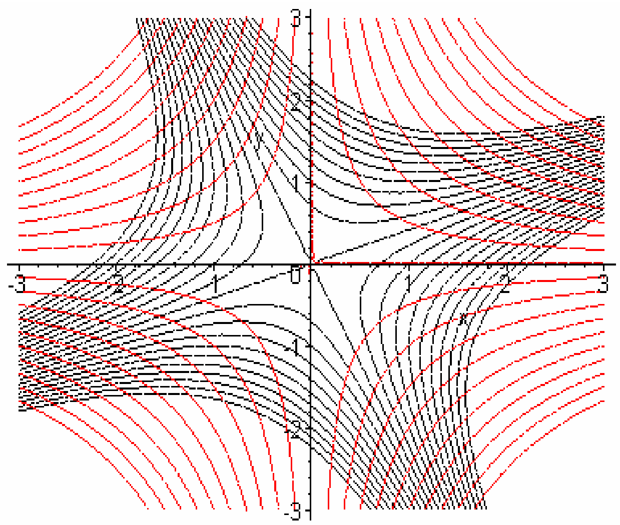
```
restart:
with(plots): a:=3:
plot1:=seq(implicitplot(x-2*y=c,
x=-a..a,y=-a..a,color=red,grid=[50,50]),c=-10..10):
plot2:=seq(implicitplot(y-3*x=c,
x=-a..a,y=-a..a,color=black,grid=[50,50]),c=-10..10):
display(plot1,plot2);
```



2) 满足与已知曲线族相交成45度角的曲线族满足如下的微分方程：

dy/dx=(x-y)/(x+y)，解得：$y^2+2xy-x^2=c$

```
restart:
with(plots): a:=3:
plot1:=seq(implicitplot(x*y=c/2,
x=-a..a,y=-a..a,color=red,grid=[50,50]),c=-10..10):
plot2:=seq(implicitplot(y^2+2*x*y-x^2=c/2,
x=-a..a,y=-a..a,color=black,grid=[50,50]),c=-10..10):
display(plot1,plot2);
```

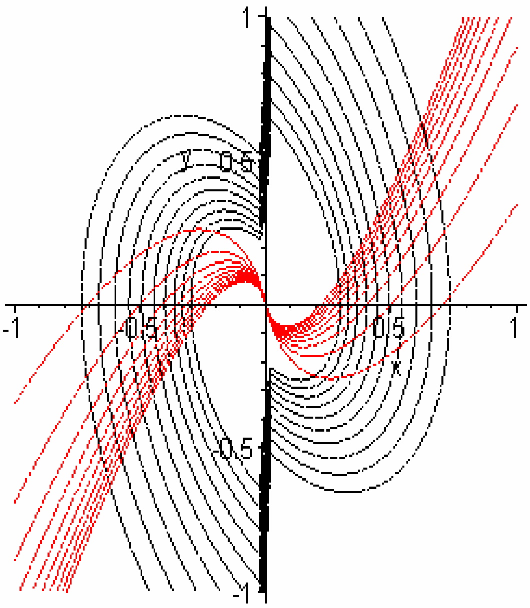


3) 满足与已知曲线族相交成45度角的曲线族满足如下的微分方程：

$y'=-\dfrac{2x+y}{y}$ 解得：

$$-\frac{1}{2}\ln\left(\frac{2x^2+yx+y^2}{x^2}\right)+\frac{1}{7}\sqrt{7}\arctan\left(\frac{1}{7}\frac{(x+2y)\sqrt{7}}{x}\right)-\ln(x)-\_C1=0$$

```
restart:
with(plots): a:=1:
plot1:=seq(implicitplot(y=x*ln(2*c*x^2)/2,
x=-a..a,y=-a..a,color=red,grid=[50,50]),c=1..10):
plot2:=seq(implicitplot(-1/2*ln((2*x^2+y*x+y^2))+1/7*sqrt
(7)*arctan(1/7*(x+2*y)*7^(1/2)/x)=c/10,
x=-a..a,y=-a..a,color=black,grid=[50,50]),c=1..10):
display(plot1,plot2);
```

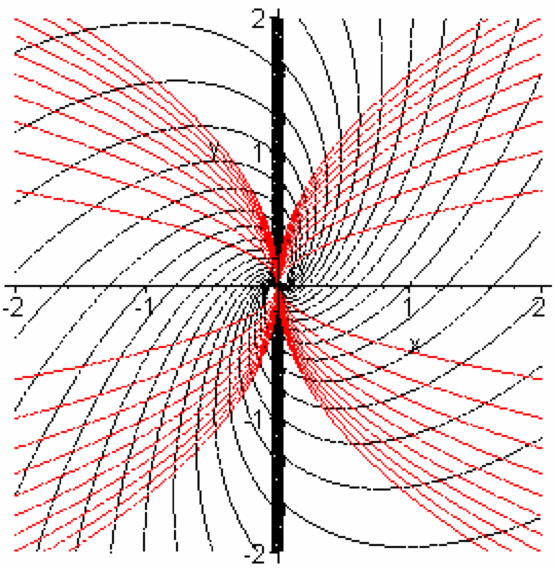


4) 满足与已知曲线族相交成45度角的曲线族满足如下的微分方程：

$y'=\dfrac{2x+y}{2x-y}$ 解得：

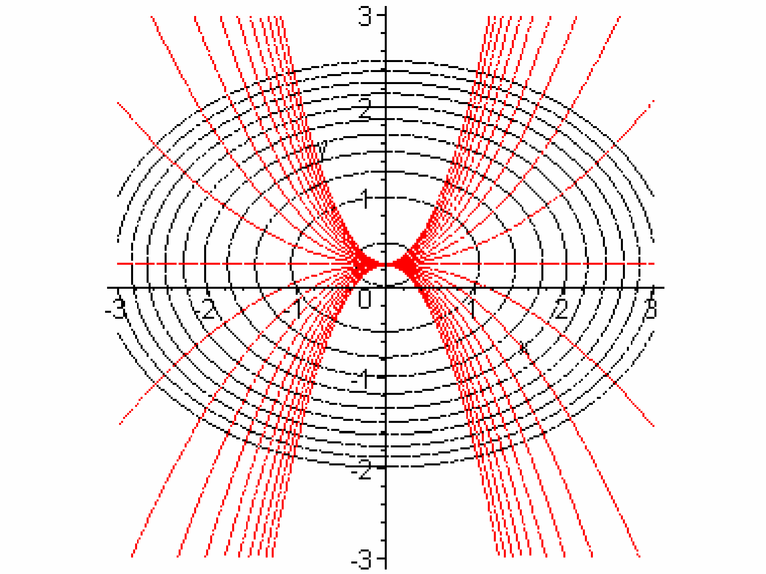
$$-\frac{1}{2}\ln\left(\frac{2x^2-yx+y^2}{x^2}\right)+\frac{3}{7}\sqrt{7}\arctan\left(\frac{1}{7}\frac{(-x+2y)\sqrt{7}}{x}\right)-\ln(x)-\_C1=0$$

```
 restart:
with(plots): a:=2:
plot1:=seq(implicitplot(y^2=c*x/4,
x=-a..a,y=-a..a,color=red,grid=[50,50]),c=-10..10):
plot2:=seq(implicitplot(-1/2*ln((2*x^2-y*x+y^2))-3/7*sqrt
(7)*arctan(1/7*(x-2*y)*7^(1/2)/x)=c/4,
x=-a..a,y=-a..a,color=black,grid=[50,50]),c=-10..10):
display(plot1,plot2);
```

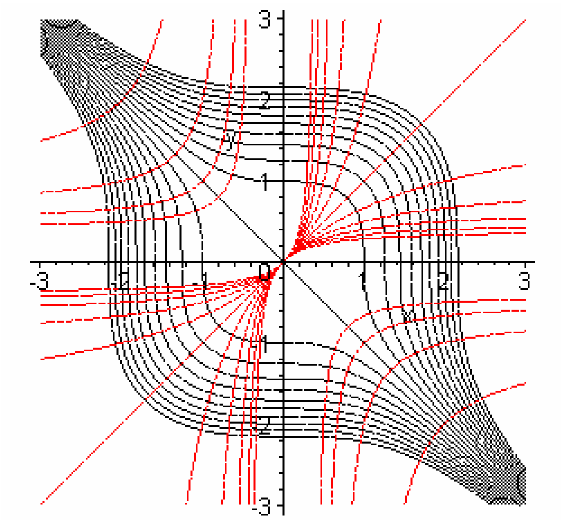


3. 抛物线族满足的微分方程：dy/dx=2c1x，dy/dx=2(y-k)/x,椭圆曲线族满足的微分方程：

dy/dx=-2x/(4y-1)；两者正交，则k=1/4。



4.两曲线族的所满足的微分方程为：$dy/dx=-y^{n-1}/x^{n-1}$，$dy/dx=x^2/y^2$，两者正交则n=3



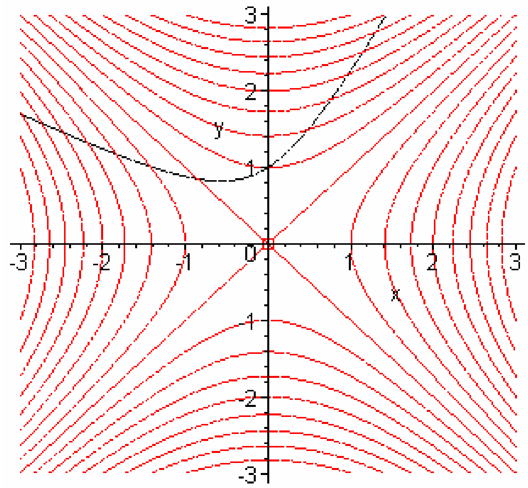
5.给定的双曲线满足微分方程dy/dx=x/y,所求得曲线满足微分方程,

$y'=\dfrac{\sqrt{3}x+y}{\sqrt{3}y-x}$，　y(0)=1,解得：

$$xy-\frac{\sqrt{3}}{2}(y^2-x^2)+\frac{\sqrt{3}}{2}=0$$

```
restart:
with(plots): a:=3:
plot1:=seq(implicitplot(x^2-y^2=c,
x=-a..a,y=-a..a,color=red,grid=[50,50]),c=-10..10):
```

```
plot2:=seq(implicitplot(x*y-3^(1/3)*(y^2-x^2-1)/2=c,
x=-a..a,y=0..a,color=black,grid=[50,50]),c=0..0):
display(plot1,plot2);
```



6.设物体下落过程中任意时刻的速度为v(t),则根据牛顿运动定律，我们得到微分方程：mdv/dt=3mg/4-kv,由物体的极限速度为24m/s，我们得到k=mg/32,v(0)=0,则我们解得：$v(t)=24(1-e^{\frac{-5t}{16}})$，v(3)=24(1-$e^{-15/16}$)＝14.6， (取 $g\approx 10$)

$s(3)=\int_0^3 24(1-e^{\frac{-5t}{16}})dt\approx 25.28$

7.设物体任意时刻的运动v(t)，根据牛顿运动定律，我们得到微分方程：

20dv/dt=10-v(t)/2,v(0)=-7,我们得到：$v(t)=20-27e^{\frac{-t}{40}}$

8. .设物体任意时刻的运动v(t)，我们设T时刻物体到达最高点，我们仅仅考虑（0,T）,根据牛顿运动定律，我们得到微分方程:10dv/dt=-10g-kv^2,根据题目中的条　件我们得到：k=2,v(0)=v_0,我们得到：$v(t)=5\sqrt{2}\tan(-\sqrt{2}t+\arctan\frac{\sqrt{2}v_0}{10})$

很容易我们就得到：v(T)=0,T=arctan($v_0\sqrt{2}$/10)/ $\sqrt{2}$.

9.设任意时刻该人在银行的存款为也y(t),y(0)=20,000 满足微分方程：

dy/dt=0.5y,我们得到：$y(t)=20000e^{0.05t}$

y(3)=23237, y(T)=40000求出T=13.9。

10.先考虑前三年，由上一题我们知道 $y(t)=5000e^{0.05t}$),

现考虑后面4年，满足微分方程：dy/dt=0.03y，

我们得到： $y(t)=5000e^{0.15}e^{0.03(t-3)}$

很容易我们就可以得到y(7)=6550

11.设任意时刻的患病老鼠的数量为N(t),则根据题目的意思，我们得到如下方程：

dN/dt=kN(5000-N),N(0)=5,解得： $N(t)=\dfrac{5000}{1+999e^{-5000kt}}$

12.设任一时刻无水部分的底面半径是 r .则 $r=\dfrac{hR}{H}$,

由体积变化的关系有：

$\dfrac{dh}{dt}=-h^{-\frac{3}{2}}\dfrac{ac\sqrt{2g}H^2}{\pi R^2}$，且 $h(0)=H$

求解得 $h(t)=\left(H^{5/2}-\dfrac{5ac\sqrt{g}H^2}{\sqrt{2}\pi R^2}t\right)^{2/5}$

当 $h=0$ 时，水放完。此时 $T=\dfrac{\pi R^2}{5ac}\sqrt{\dfrac{2H}{g}}$