答案 5.2

1．解：　解曲线是：$x = x_0 \cos t \qquad y = x_0 \sin t$

相平面的轨线是：$x^2 + y^2 = x_0^2$

2．解：　由方程组 1）$\dfrac{dx}{dy} = \dfrac{x}{y}$，　$x = cy$，过点（1，2）$\therefore$　$y = 2x$，由非自治系统

$$\frac{dx}{dt} - x - t = 0 \qquad x = c_1 e^t - t - 1$$
$$\frac{dy}{dt} = y \qquad y = c_2 e^t$$

过点（1，2）$x = (2 + t_0)e^{-t_0 + t} - t - 1,\quad y = 2e^{-t_0 + t}$

即　$x = (1 + \dfrac{t_0}{2})y - t_0 + \ln 2 - \ln y - 1$

3．证明：$\dfrac{dx}{dy} = \dfrac{x}{y}$　　$\therefore$　$x = cy \quad x_0 = cy_0 \quad xy_0 = yx_0$，

当 $t = t_0$ 时，从 (x_0, y_0) 出发的解与初始时刻 t_0 无关。其原因是由于右端有相同的因子，可以通过时间变换将其变化为自治系统。

4．证明: 1) 性质 4:

设有两条轨线 $x = x(t),\ y = y(t)$ 及 $x = x(t+T),\ y = y(t+T)$　$\exists t_0, \exists T > 0$ 有

$$x = x(t_0) = x(t_0 + T),\quad y = y(t_0) = y(t_0 + T)$$

$\therefore$ 两条轨线交于同一点 $(x(t_0), y(t_0))$,由性质 2 可知. 由解的存在唯一性定理.

过相平面上 $\forall$ 一点 (x_0, y_0) 系统有且仅有一条轨线通过.

$$\therefore \quad x(t) = x(t+T) \quad y(t) = y(t+T)$$

2) 性质 5：(反证法)

若系统出发于 $\forall$ 非奇点的轨线在有限时间内达到某奇点 (x^*, y^*)，即有一轨线过点 (x^*, y^*)，而系统又 $\exists$ 一条常轨线 $x = x^*, y = y^*$,

由解的 $\exists$ 唯一性定理, 两条轨线不可能交于同一点 $x = x^*, y = y^*$.

$\therefore$ 性质 5 得证.

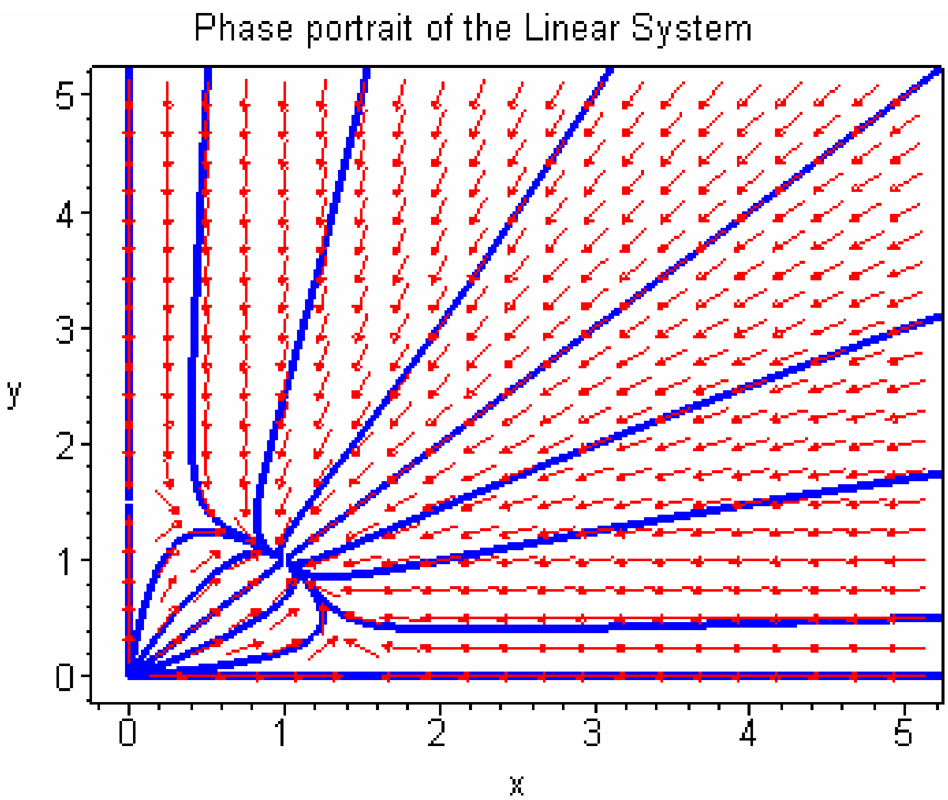
5 解:　由方程可得,　该方程组有 4 个平衡点

$$0(0,0) \quad A(0,\frac{r_2}{\beta_2}) \quad B(\frac{r_1}{\alpha_1},0) \quad C\left(\frac{\beta_2 r_1-\alpha_2 r_2}{\alpha_1\beta_2-\alpha_2\beta_1},\ \frac{\alpha_1 r_2-\beta_1 r_1}{\beta_2\alpha_1-\alpha_2\beta_1}\right)$$

解轨线的走向请参阅例 5.2.3 和下面的图形

```
restart:with(DEtools):
ODE1 :=[diff(x(t),t)=x(t)*(3-2*x(t)-y(t)),
diff(y(t),t)=y(t)*(3-x(t)-2*y(t))];
DEplot( ODE1, [x(t),y(t)],t=-100..100,
[[x(0)=0,y(0)=1],[x(0)=0,y(0)=5],
[x(0)=0.1,y(0)=0.6],[x(0)=0.6,y(0)=0.1],
[x(0)=5,y(0)=0],[x(0)=2,y(0)=1],[x(0)=0.1,y(0)=0.1],
[x(0)=5,y(0)=5], [x(0)=1.5,y(0)=5],[x(0)=0.1,y(0)=0.2],
[x(0)=0.2,y(0)=0.1],[x(0)=1,y(0)=0],[x(0)=5,y(0)=3],
[x(0)=5,y(0)=0.5],[x(0)=0.5,y(0)=5],[x(0)=3,y(0)=5]],
 x=0..5,y=0..5,stepsize=0.05, dirgrid=[21,21],
 color=red, linecolor=blue,axes=BOXED,
 title="Phase portrait of the Linear System",
arrows=SLIM);
```

$$ODE1 := \left[\frac{\partial}{\partial t}\mathrm{x}(t)=\mathrm{x}(t)\,(3-2\,\mathrm{x}(t)-\mathrm{y}(t)),\ \frac{\partial}{\partial t}\mathrm{y}(t)=\mathrm{y}(t)\,(3-\mathrm{x}(t)-2\,\mathrm{y}(t))\right]$$



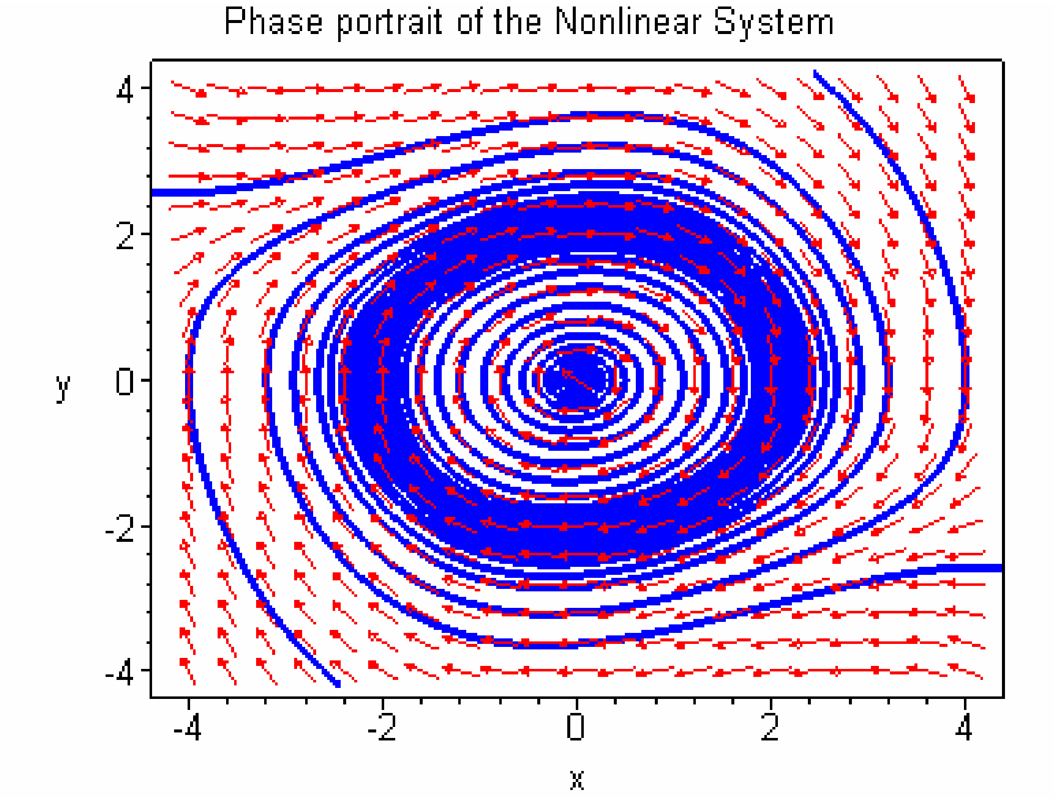
6．对不同的参数值所得到的解轨线如下

```
restart:with(DEtools): mu:=0.1; a:=4;
ODE1 :=[diff(x(t),t)=y(t),
diff(y(t),t)=-x(t)-mu*y(t)*(x(t)^2-1)];
DEplot( ODE1, [x(t),y(t)],t=-100..100,
[[x(0)=0.1,y(0)=0.1],[x(0)=0,y(0)=0.8*a],
[x(0)=0,y(0)=-0.8*a],
[x(0)=-0.8*a,y(0)=0],[x(0)=0.8*a,y(0)=0]],
 x=-a..a,y=-a..a,stepsize=0.05, dirgrid=[21,21],
 color=red, linecolor=blue,axes=BOXED,
 title="Phase portrait of the Nonlinear System",
arrows=SLIM);
```

$$\mu := .1$$

$$a := 4$$

$$ODE1 := \left[\frac{\partial}{\partial t}\,\mathrm{x}(t) = \mathrm{y}(t), \frac{\partial}{\partial t}\,\mathrm{y}(t) = -\mathrm{x}(t) - .1\,\mathrm{y}(t)\,(\mathrm{x}(t)^2 - 1)\right]$$



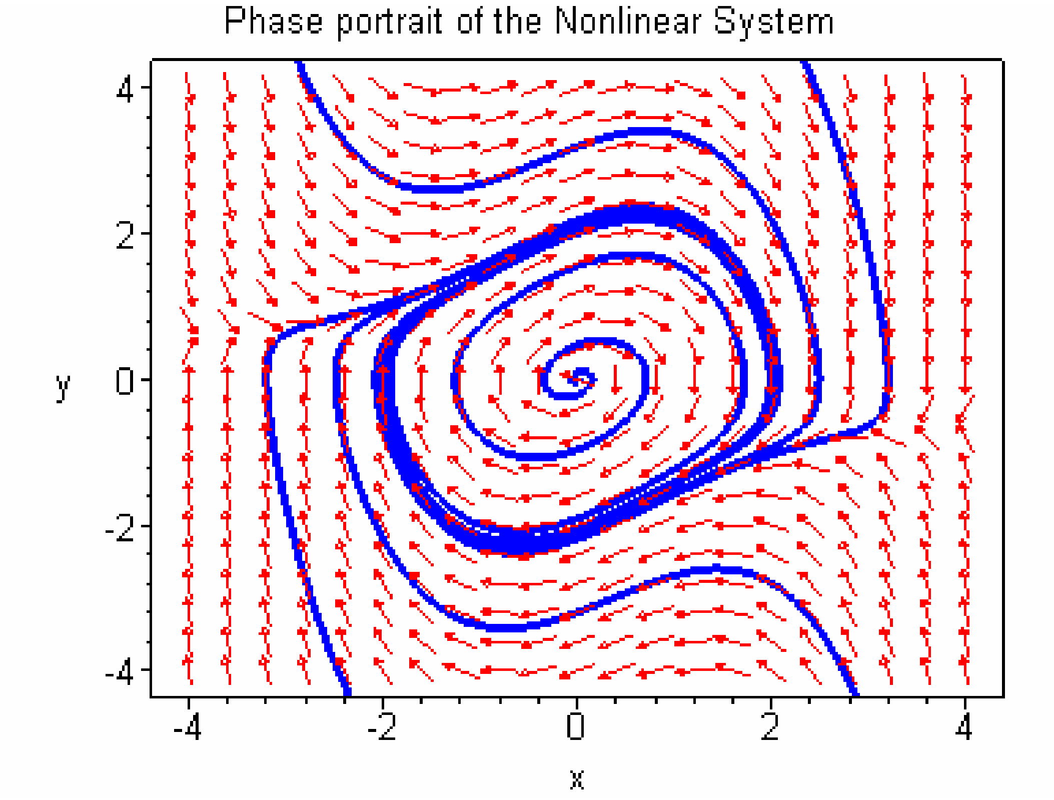

```
 restart:with(DEtools): mu:=0.5; a:=4;
ODE1 :=[diff(x(t),t)=y(t),
diff(y(t),t)=-x(t)-mu*y(t)*(x(t)^2-1)];
DEplot( ODE1, [x(t),y(t)],t=-100..100,
[[x(0)=0.1,y(0)=0.1],[x(0)=0,y(0)=0.8*a],
[x(0)=0,y(0)=-0.8*a],
[x(0)=-0.8*a,y(0)=0],[x(0)=0.8*a,y(0)=0]],
```

```
 x=-a..a,y=-a..a,stepsize=0.05, dirgrid=[21,21],
 color=red, linecolor=blue,axes=BOXED,
 title="Phase portrait of the Nonlinear System",
arrows=SLIM);
```

$$\mu := .5$$

$$a := 4$$

$$ODE1 := \left[\frac{\partial}{\partial t}\,\mathrm{x}(t) = \mathrm{y}(t), \frac{\partial}{\partial t}\,\mathrm{y}(t) = -\mathrm{x}(t) - .5\,\mathrm{y}(t)\,(\mathrm{x}(t)^2 - 1)\right]$$

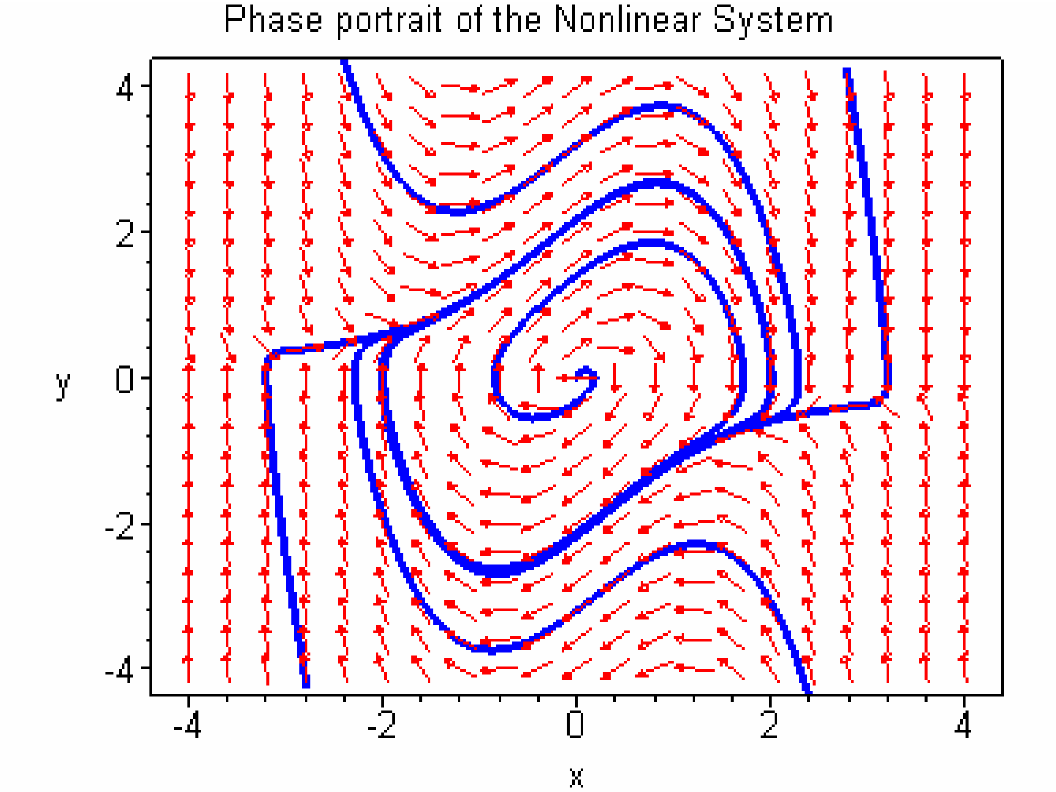



```
> restart:with(DEtools): mu:=1; a:=4;
ODE1 :=[diff(x(t),t)=y(t),
diff(y(t),t)=-x(t)-mu*y(t)*(x(t)^2-1)];
DEplot( ODE1, [x(t),y(t)],t=-100..100,
[[x(0)=0.1,y(0)=0.1],[x(0)=0,y(0)=0.8*a],
[x(0)=0,y(0)=-0.8*a],
[x(0)=-0.8*a,y(0)=0],[x(0)=0.8*a,y(0)=0]],
 x=-a..a,y=-a..a,stepsize=0.05, dirgrid=[21,21],
 color=red, linecolor=blue,axes=BOXED,
 title="Phase portrait of the Nonlinear System",
arrows=SLIM);
```

$$\mu := 1$$

$$a := 4$$

$$ODE1 := \left[\frac{\partial}{\partial t}\,x(t) = y(t),\ \frac{\partial}{\partial t}\,y(t) = -x(t) - y(t)\,(x(t)^2 - 1)\right]$$

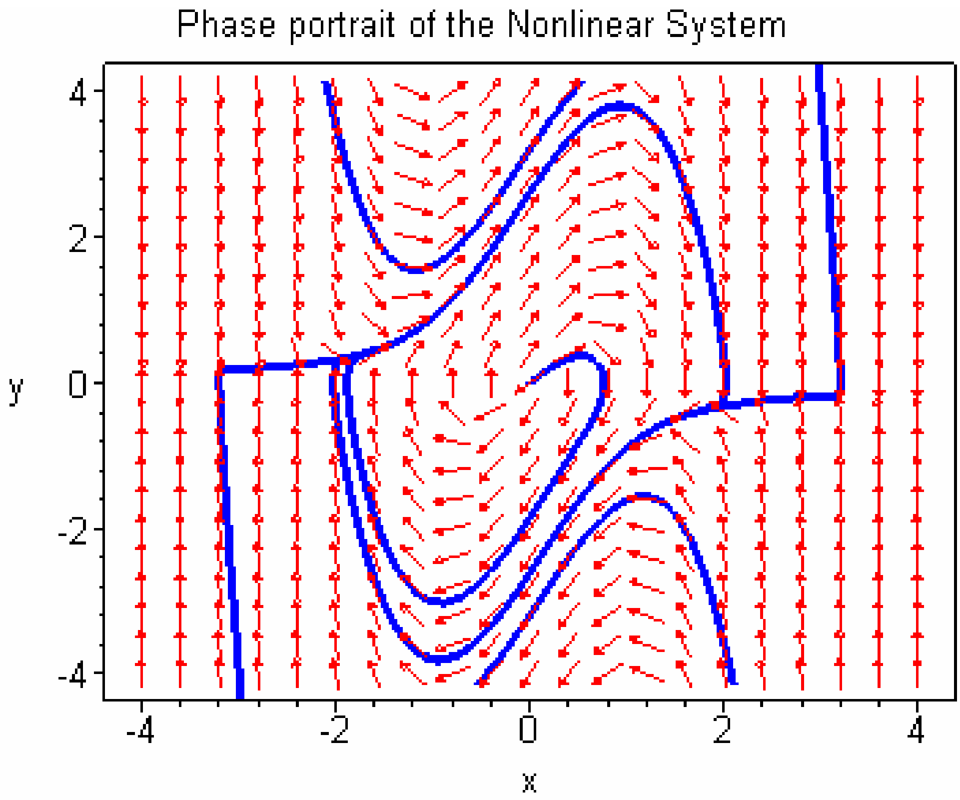



```
> restart:with(DEtools): mu:=2; a:=4;
ODE1 :=[diff(x(t),t)=y(t),
diff(y(t),t)=-x(t)-mu*y(t)*(x(t)^2-1)];
DEplot( ODE1, [x(t),y(t)],t=-100..100,
[[x(0)=0.1,y(0)=0.1],[x(0)=0,y(0)=0.8*a],
[x(0)=0,y(0)=-0.8*a],
[x(0)=-0.8*a,y(0)=0],[x(0)=0.8*a,y(0)=0]],
 x=-a..a,y=-a..a,stepsize=0.05, dirgrid=[21,21],
 color=red, linecolor=blue,axes=BOXED,
 title="Phase portrait of the Nonlinear System",
arrows=SLIM);
```

$$\mu := 2$$

$$a := 4$$

$$ODE1 := \left[\frac{\partial}{\partial t}\,x(t) = y(t),\ \frac{\partial}{\partial t}\,y(t) = -x(t) - 2\,y(t)\,(x(t)^2 - 1)\right]$$


Phase portrait of the Nonlinear System
y
4
2
0
-2
-4
-4
-2
0
2
4
x