

答案 2.6

1.

(1)

```
restart: n:=5:
x0:=0: y0:=1: y[0]:=y0;
f:=(x,y)->x*y;
for j from 1 to n do
y[j]:=y0+int(f(x,y[j-1]),x=x0..x);
end do;
```

$$y_0 := 1$$

$$f := (x, y) \rightarrow x y$$

$$y_1 := 1 + \frac{1}{2} x^2$$

$$y_2 := 1 + \frac{1}{8} x^4 + \frac{1}{2} x^2$$

$$y_3 := 1 + \frac{1}{48} x^6 + \frac{1}{8} x^4 + \frac{1}{2} x^2$$

$$y_4 := 1 + \frac{1}{384} x^8 + \frac{1}{48} x^6 + \frac{1}{8} x^4 + \frac{1}{2} x^2$$

$$y_5 := 1 + \frac{1}{3840} x^{10} + \frac{1}{384} x^8 + \frac{1}{48} x^6 + \frac{1}{8} x^4 + \frac{1}{2} x^2$$

(2) > restart: n:=5:

```
x0:=0: y0:=1: y[0]:=y0;
f:=(x,y)->x+y;
for j from 1 to n do
y[j]:=y0+int(f(x,y[j-1]),x=x0..x);
end do;
```

$$y_0 := 1$$

$$f := (x, y) \rightarrow x + y$$

$$y_1 := 1 + \frac{1}{2} x^2 + x$$

$$y_2 := 1 + x^2 + x + \frac{1}{6} x^3$$

$$y_3 := 1 + x^2 + x + \frac{1}{3} x^3 + \frac{1}{24} x^4$$

$$y_4 := 1 + x^2 + x + \frac{1}{3} x^3 + \frac{1}{12} x^4 + \frac{1}{120} x^5$$

$$y_5 := 1 + x^2 + x + \frac{1}{3}x^3 + \frac{1}{12}x^4 + \frac{1}{60}x^5 + \frac{1}{720}x^6$$

```
> restart: n:=5:
(3)
x0:=0: y0:=0: y[0]:=y0;
f:=(x,y)->x+y^2;
for j from 1 to n do
y[j]:=y0+int(f(x,y[j-1]),x=x0..x);
end do;
```

$$y_0 := 0$$

$$f := (x, y) \rightarrow x + y^2$$

$$y_1 := \frac{1}{2}x^2$$

$$y_2 := \frac{1}{2}x^2 + \frac{1}{20}x^5$$

$$y_3 := \frac{1}{4400}x^{11} + \frac{1}{160}x^8 + \frac{1}{20}x^5 + \frac{1}{2}x^2$$

$$y_4 := \frac{1}{445280000}x^{23} + \frac{1}{7040000}x^{20} + \frac{87}{23936000}x^{17} + \frac{3}{49280}x^{14} + \frac{7}{8800}x^{11} + \frac{1}{160}x^8 + \frac{1}{20}x^5 + \frac{1}{2}x^2$$

$$y_5 := \frac{1}{93188910848000000000}x^{47} + \frac{1}{689649664000000000}x^{44} + \frac{7781}{87397421056000000000}x^{41} + \frac{194877}{56701741465600000000}x^{38} + \frac{6915599}{71026391941120000000}x^{35} + \frac{236231}{108520079360000000}x^{32} + \frac{1363623}{3442121267200000}x^{29} + \frac{384503}{62980403200000}x^{26} + \frac{8751}{105976640000}x^{23} + \frac{659}{670208000}x^{20} + \frac{1769}{167552000}x^{17} + \frac{1}{9856}x^{14} + \frac{7}{8800}x^{11} + \frac{1}{160}x^8 + \frac{1}{20}x^5 + \frac{1}{2}x^2$$

```
(4)
restart: n:=5:
x0:=0: y0:=0: y[0]:=y0;
f:=(x,y)->1+x*y^2;
for j from 1 to n do
y[j]:=y0+int(f(x,y[j-1]),x=x0..x);
end do;
```

$$y_0 := 0$$

$$f := (x, y) \rightarrow 1 + x y^2$$

$$y_1 := x$$

$$y_2 := x + \frac{1}{4}x^4$$

$$y_3 := \frac{1}{160}x^{10} + \frac{1}{14}x^7 + \frac{1}{4}x^4 + x$$

$$y_4 := \frac{1}{563200}x^{22} + \frac{1}{21280}x^{19} + \frac{129}{250880}x^{16} + \frac{27}{7280}x^{13} + \frac{23}{1120}x^{10} + \frac{1}{14}x^7 + \frac{1}{4}x^4 + x$$

$$\begin{aligned} y_5 := & \frac{1}{14590935040000}x^{46} + \frac{1}{257675264000}x^{43} + \frac{102889}{1020154347520000}x^{40} \\ & + \frac{29343}{17654500864000}x^{37} + \frac{586468081}{29071714287616000}x^{34} + \frac{1144727}{5916643532800}x^{31} \\ & + \frac{210773907}{138945693286400}x^{28} + \frac{3015919}{298217920000}x^{25} + \frac{807201}{13632819200}x^{22} + \frac{271}{885248}x^{19} \\ & + \frac{929}{652288}x^{16} + \frac{43}{7280}x^{13} + \frac{23}{1120}x^{10} + \frac{1}{14}x^7 + \frac{1}{4}x^4 + x \end{aligned}$$

(5)

```
restart: n:=4:
x0:=0: y0:=0: y[0]:=y0;
f:=(x,y)->exp(x)+y^2;
for j from 1 to n do
y[j]:=subs(t=x,y0+int(f(x,y[j-1]),x=x0..t));
end do;
```

$$y_0 := 0$$

$$f := (x, y) \rightarrow e^x + y^2$$

$$y_1 := e^x - 1$$

$$y_2 := -e^x + \frac{1}{2}(e^x)^2 + \ln(e^x) + \frac{1}{2}$$

$$\begin{aligned} y_3 := & 2e^x + \frac{1}{2}(e^x)^2 - \frac{1}{3}(e^x)^3 - 2e^x \ln(e^x) + \frac{1}{16}(e^x)^4 + \frac{1}{2}(e^x)^2 \ln(e^x) + \frac{1}{3}\ln(e^x)^3 \\ & + \frac{1}{2}\ln(e^x)^2 + \frac{1}{4}\ln(e^x) - \frac{107}{48} \end{aligned}$$

$$\begin{aligned} y_4 := & \frac{67}{162}(e^x)^3 \ln(e^x) + \frac{9}{4}(e^x)^2 \ln(e^x)^2 - \frac{4}{3}e^x \ln(e^x)^4 + \frac{1}{18}\ln(e^x)^6 - \frac{89}{288}\ln(e^x)^4 \\ & + \frac{1}{12}(e^x)^2 \ln(e^x)^3 + \frac{331}{768}(e^x)^4 \ln(e^x) + \frac{1}{6}(e^x)^2 \ln(e^x)^4 - \frac{19}{27}(e^x)^3 \ln(e^x)^2 \end{aligned}$$

$$\begin{aligned}
& -\frac{2}{27}(\mathbf{e}^x)^3 \ln(\mathbf{e}^x)^3 + \frac{1}{12} \ln(\mathbf{e}^x)^5 - 13 \mathbf{e}^x \ln(\mathbf{e}^x)^2 - \frac{7}{60}(\mathbf{e}^x)^5 \ln(\mathbf{e}^x) + \frac{14}{3} \mathbf{e}^x \ln(\mathbf{e}^x)^3 \\
& + \frac{1}{96}(\mathbf{e}^x)^4 \ln(\mathbf{e}^x)^3 + \frac{9}{128}(\mathbf{e}^x)^4 \ln(\mathbf{e}^x)^2 + \frac{1}{96}(\mathbf{e}^x)^6 \ln(\mathbf{e}^x) + \frac{1}{150}(\mathbf{e}^x)^5 + \frac{47}{1728}(\mathbf{e}^x)^6 \\
& + \frac{1}{2048}(\mathbf{e}^x)^8 - \frac{1}{168}(\mathbf{e}^x)^7 + \frac{865}{192}(\mathbf{e}^x)^2 + \frac{11449}{2304} \ln(\mathbf{e}^x) + \frac{1}{63} \ln(\mathbf{e}^x)^7 + \frac{3372501217}{87091200} \\
& + \frac{431}{12} \mathbf{e}^x \ln(\mathbf{e}^x) - \frac{695}{96}(\mathbf{e}^x)^2 \ln(\mathbf{e}^x) + \frac{1991}{1944}(\mathbf{e}^x)^3 - \frac{459}{1024}(\mathbf{e}^x)^4 - \frac{107}{192} \ln(\mathbf{e}^x)^2 \\
& - \frac{13}{18} \ln(\mathbf{e}^x)^3 - \frac{263}{6} \mathbf{e}^x
\end{aligned}$$

(6)

```

restart: n:=3:
x0:=0: y0:=0: y[0]:=y0;
f:=(x,y)->sin(x)+y^2;
for j from 1 to n do
y[j]:=subs(t=x,y0+int(f(x,y[j-1]),x=x0..t));
end do;

```

$$y_0 := 0$$

$$f := (x, y) \rightarrow \sin(x) + y^2$$

$$y_1 := -\cos(x) + 1$$

$$y_2 := -\cos(x) + \frac{3}{2}x - 2 \sin(x) + \frac{1}{2} \cos(x) \sin(x) + 1$$

$$\begin{aligned}
y_3 := & \frac{125}{32}x + \frac{3}{4}x^3 + \frac{3}{2}x^2 - \frac{35}{32} \cos(x) \sin(x) - \frac{1}{16} \sin(x) \cos(x)^3 - \frac{2}{3} \sin(x)^3 - 8 \sin(x) \\
& - \frac{5}{2} \cos(x)^2 - 3x \sin(x) + \frac{1}{3} \cos(x)^3 + 6x \cos(x) - \frac{3}{4}x \cos(x)^2 + \frac{13}{6}
\end{aligned}$$

(7)

```

> restart: n:=4:
x0:=0: y0:=0: y[0]:=y0;
f:=(x,y)->2*x+y^3;
for j from 1 to n do
y[j]:=subs(t=x,y0+int(f(x,y[j-1]),x=x0..t));
end do;

```

$$y_0 := 0$$

$$f := (x, y) \rightarrow 2x + y^3$$

$$y_1 := x^2$$

$$y_2 := x^2 + \frac{1}{7}x^7$$

$$y_3 := \frac{1}{7546} x^{22} + \frac{3}{833} x^{17} + \frac{1}{28} x^{12} + \frac{1}{7} x^7 + x^2$$

$$\begin{aligned} y_4 := & \frac{1}{28788908891512} x^{67} + \frac{9}{2940832522936} x^{62} + \frac{1081}{8754736450768} x^{57} \\ & + \frac{76623}{25457852047628} x^{52} + \frac{310503}{6187558093024} x^{47} + \frac{1069}{1709742496} x^{42} \\ & + \frac{790667}{126520944704} x^{37} + \frac{11871}{229881344} x^{32} + \frac{361}{1026256} x^{27} + \frac{47}{23324} x^{22} + \frac{33}{3332} x^{17} \\ & + \frac{1}{28} x^{12} + \frac{1}{7} x^7 + x^2 \end{aligned}$$

(8)

```
> restart: n:=4:
  x0:=0: y0:=0: y[0]:=y0;
  f:=(x,y)->1+6*x*y^4;
  for j from 1 to n do
  y[j]:=subs(t=x,y0+int(f(x,y[j-1]),x=x0..t));
  end do;
```

$$y_0 := 0$$

$$f := (x, y) \rightarrow 1 + 6 x y^4$$

$$y_1 := x$$

$$y_2 := x + x^6$$

$$y_3 := \frac{3}{13} x^{26} + \frac{8}{7} x^{21} + \frac{9}{4} x^{16} + \frac{24}{11} x^{11} + x^6 + x$$

$$\begin{aligned} y_4 := & \frac{243}{1513733} x^{106} + \frac{5184}{1553279} x^{101} + \frac{56835}{1722448} x^{96} + \frac{155594160}{754324571} x^{91} + \frac{18249105135}{19960588648} x^{86} \\ & + \frac{76011956}{24867843} x^{81} + \frac{34204004451}{4264516256} x^{76} + \frac{651748161}{38307269} x^{71} + \frac{590921089175}{19751443712} x^{66} \\ & + \frac{59994903015}{1344685342} x^{61} + \frac{284103223641}{4937860928} x^{56} + \frac{441198687}{6782776} x^{51} + \frac{111491759853}{1716042328} x^{46} \\ & + \frac{81770511}{1418846} x^{41} + \frac{12629135}{276848} x^{36} + \frac{10993230}{341341} x^{31} + \frac{221868}{11011} x^{26} + \frac{862}{77} x^{21} + \frac{243}{44} x^{16} \\ & + \frac{24}{11} x^{11} + x^6 + x \end{aligned}$$

2.

1) 可以用下面的 Maple 语句求解

```
ode1:=diff(y(x),x)=x+y(x);
ans1:=dsolve({ode1,y(0)=1},y(x),type=series);
```

结果为

$$ode1 := \frac{\partial}{\partial x} y(x) = x + y(x)$$

$$ans1 := y(x) = 1 + x + x^2 + \frac{1}{3}x^3 + \frac{1}{12}x^4 + \frac{1}{60}x^5 + O(x^6)$$

(2)

```
> restart: Order:=10:
ode1:=diff(y(x),x)=x^2+2*y^2;
ans1:=dsolve({ode1,y(0)=4},y(x),type=series);
```

$$ode1 := \frac{\partial}{\partial x} y(x) = x^2 + 2 y^2$$

$$ans1 := y(x) = 4 + 32 x + 256 x^2 + \frac{6145}{3} x^3 + \frac{49156}{3} x^4 + \frac{655424}{5} x^5 + \frac{15730432}{15} x^6 + \frac{2642755594}{315} x^7 + \frac{2349154314}{35} x^8 + \frac{169141863424}{315} x^9 + O(x^{10})$$

(3)

```
restart: Order:=10:
ode1:=diff(y(x),x)=x+sin(y);
ans1:=dsolve({ode1,y(0)=0},y(x),type=series);
```

$$ode1 := \frac{\partial}{\partial x} y(x) = x + \sin(y)$$

ans1 :=

$$y(x) = \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 + \frac{1}{720}x^6 - \frac{1}{360}x^7 - \frac{17}{5760}x^8 - \frac{29}{17280}x^9 + O(x^{10})$$

(4)

```
restart: Order:=10:
ode1:=diff(y(x),x)=1+x*sin(y);
ans1:=dsolve({ode1,y(0)=0},y(x),type=series);
```

$$ode1 := \frac{\partial}{\partial x} y(x) = 1 + x \sin(y)$$

$$ans1 := y(x) = x + \frac{1}{3}x^3 + \frac{1}{30}x^5 - \frac{1}{56}x^7 - \frac{11}{1296}x^9 + O(x^{10})$$

(5)

```
restart: Order:=10:
ode1:=diff(y(x),x)=exp(x)+x*cos(y);
ans1:=dsolve({ode1,y(0)=0},y(x),type=series);
```

$$ode1 := \frac{\partial}{\partial x} y(x) = e^x + x \cos(y)$$

$$ans1 := y(x) =$$

$$x + x^2 + \frac{1}{6}x^3 - \frac{1}{12}x^4 - \frac{23}{120}x^5 - \frac{37}{360}x^6 + \frac{61}{5040}x^7 + \frac{271}{4032}x^8 + \frac{21617}{362880}x^9 + O(x^{10})$$

3.

$$1) y = 1 + 2(x-1) + \frac{7}{2}(x-1)^2 + \frac{14}{3}(x-1)^3$$

$$2) y = 1 + 3(x-1) + 5(x-1)^2 + 8(x-1)^3$$

$$3) \pi + \frac{1}{2}(x-1)^2 + \frac{1}{40}(x-1)^5$$

$$4) \frac{1}{2}\pi + 2x - 2 + \frac{3}{2}(x-1)^2 - \frac{1}{3}(x-1)^3 - \frac{5}{4}(x-1)^4 - \frac{67}{120}(x-1)^5$$

4.

参阅以下程序和结果，其中第一列数据为自变量的坐标，第二列为Euler折线法计算的结果，第三列为改进的Euler折线法计算结果，第四列为精确解的值

```
(1) > restart: printlev1:=0:
h:=0.1: x0:=0: y0:=1: z0:=1: s0:=1:
f1:=(x,y)->-2*y+x^3*exp(-2*x):
f2:=(x,y)->-2*(-2*y+x^3*exp(-2*x))+(3*x^2-2*x^3)*exp(-2*x):
f3:=x->(1/4*x^4+1)*exp(-2*x):
for n from 0 to 9 do;
x|(n+1):=h*(n+1);
y|(n+1):=y|n+h*f1(x|n,y|n);
z|(n+1):=z|n+h*f1(x|n,z|n)+h^2*f2(x|n,z|n)/2;
s|(n+1):=f3(x|(n+1));
print(x|(n+1),y|(n+1),z|(n+1),s|(n+1));od:
```

.1, .8, .8200000000, .8187512214

.2, .6400818731, .6725883081, .6705881740

.3, .5126017545, .5523536095, .5499229797

.4, .4115631950, .4548562886, .4522046695

.5, .3321262614, .3763611104, .3736275575

.6, .2702995021, .3136744528, .3109529044

.7, .2227453967, .2640441360, .2613989467

.8, .1866545932, .2250952999, .2225707214

.9, .1596607763, .1947860338, .1924120383

1.0, .1397789100, .1713731604, .1691691040

(2)

```
> restart: printlevl:=0:
h:=0.1: x0:=0: y0:=1: z0:=1: s0:=1:
f1:=(x,y)->4*y-2*y^2:
f2:=(x,y)->4*(1-y)*(4*y-2*y^2):
f3:=x->2/(1+exp(-4*x)):
for n from 0 to 9 do;
x||(n+1):=h*(n+1);
y||(n+1):=y||n+h*f1(x||n,y||n);
z||(n+1):=z||n+h*f1(x||n,z||n)+h^2*f2(x||n,z||n)/2;
s||(n+1):=f3(x||n+1);
print(x||(n+1),y||(n+1),z||(n+1),s||(n+1));od:
```

.1, 1.2, 1.2, 1.197375320

.2, 1.392, 1.384320000, 1.379948962

.3, 1.5612672, 1.541677419, 1.537049567

.4, 1.698263026, 1.667684876, 1.664036770

.5, 1.800748775, 1.763723101, 1.761594156

.6, 1.872509055, 1.834337944, 1.833654607

.7, 1.920254645, 1.884972432, 1.885351648

.8, 1.950880923, 1.920661868, 1.921668554

.9, 1.970046017, 1.945526531, 1.946806014

1.0, 1.981848162, 1.962714187, 1.964027580

5.

(1) 程序和结果如下，其中第一列数据为自变量的坐标，第二列为Euler折线法计算的结果，第三列为改进的Euler折线法计算结果，第四列为精确解的值

```
> restart: printlevl:=0:
h:=0.1: x0:=0: y0:=2: z0:=2: s0:=2:
f1:=(x,y)->-3*y+7*exp(4*x):
f2:=(x,y)->28*exp(4*x)-3*(-3*y+7*exp(4*x)):
f3:=x->(exp(7*x)+1)*exp(-3*x):
for n from 0 to 9 do;
x||(n+1):=h*(n+1);
y||(n+1):=y||n+h*f1(x||n,y||n);
z||(n+1):=z||n+h*f1(x||n,z||n)+h^2*f2(x||n,z||n)/2;
s||(n+1):=f3(x||n+1);
```

```
print(x||(n+1),y||(n+1),z||(n+1),s||(n+1));od:
```

```
.1, 2.1, 2.225000000, 2.232642918
.2, 2.514277289, 2.754116153, 2.774352565
.3, 3.317872752, 3.687589116, 3.726686582
.4, 4.646592772, 5.187539830, 5.254226636
.5, 6.719737637, 7.505196005, 7.612186258
.6, 9.876155615, 11.02232725, 11.18847527
.7, 14.62953240, 16.31366844, 16.56710321
.8, 21.75192542, 24.24049837, 24.62324815
.9, 32.39911893, 36.09058098, 36.66543995
1.0, 48.29814736, 53.78718515, 54.64793708
```

(2) 程序和结果如下，其中第一列数据为自变量的坐标，第二列为Euler折线法计算的结果，第三列为改进的Euler折线法计算结果

```
restart: printlevl:=0:
h:=0.1: x0:=1: y0:=3.1415926: z0:=3.1415926:
f1:=(x,y)->-x^2*y+sin(x*y):
f2:=(x,y)->-2*x*y+y*cos(x*y)
      +(-x^2+x*cos(x*y))*(-x^2*y+sin(x*y)):
for n from 0 to 9 do;
x||(n+1):=h*(n+1);
y||(n+1):=y||n+h*f1(x||n,y||n);
z||(n+1):=z||n+h*f1(x||n,z||n)+h^2*f2(x||n,z||n)/2;
print(x||(n+1),y||(n+1),z||(n+1));od:
```

```
.1, 2.827433345, 2.811725381
.2, 2.852505022, 2.847464037
.3, 2.895100380, 2.896563180
.4, 2.945382506, 2.947781855
.5, 2.990646473, 2.986862965
.6, 3.015595637, 2.997847693
.7, 3.004202095, 2.966225762
.8, 2.943168257, 2.882822786
.9, 2.825633441, 2.746233380
1.0, 2.653099379, 2.562556615
```

(3) 程序和结果如下，其中第一列数据为自变量的坐标，第二列为Euler折线法计算的结果，第三列为改进的Euler折线法计算结果

```
restart: printlevel:=0:
h:=0.1: x0:=0: y0:=2: z0:=2:
f1:=(x,y)->x^2+y^2-4:
f2:=(x,y)->2*x+2*y*(x^2+y^2-4):
for n from 0 to 9 do;
x||(n+1):=h*(n+1);
y||(n+1):=y| |n+h*f1(x| |n,y| |n);
z| |(n+1):=z| |n+h*f1(x| |n,z| |n)+h^2*f2(x| |n,z| |n)/2;
print(x| |(n+1),y| |(n+1),z| |(n+1));od:
```

```
.1, 2., 2.
.2, 2.001, 2.002200000
.3, 2.0054001, 2.010057655
.4, 2.016563056, 2.028710577
.5, 2.039215712, 2.065869716
.6, 2.080055784, 2.133348923
.7, 2.148718990, 2.249905284
.8, 2.259418320, 2.447032853
.9, 2.433915434, 2.782137115
1.0, 2.707309868, 3.372761029
```

注意，由于左端函数的不同，数值计算的精度有差异，必要时需调整步长或用更精确的算法。