

习题 2.5.

1.

$$1) \text{ 令 } y' = \frac{1}{t}; x = t^3 + t^2, y = \frac{3}{2}t^2 + 2t + c$$

$$2) y = -x^2 \quad \text{或} \quad y = -\frac{1}{2}x^2 + Cx + \frac{C^2}{2},$$

$$3) y = -\frac{1}{4}(\ln x)^2 \quad \text{或} \quad y = c \ln x + c^2$$

$$4) y = \pm 2x \quad \text{或} \quad y = \frac{c}{2}x^2 + \frac{2}{c}$$

$$5) y = \sqrt{1-x^2}, \quad \text{或} \quad y = cx + \sqrt{1+c^2}$$

2.

$$1) \text{ 令 } y' = \tan \phi; \quad y = a(1 + \cos 2\phi), \quad x = -a(2\phi + \sin 2\phi) + c; \quad y = 2a$$

$$2) \text{ 令 } p = ty, \text{ 则 } y = \pm\sqrt{2}, \quad y = 2(5t^2 + 2)^{-1/2}, x = -\sqrt{\frac{5}{2}} \arctan \sqrt{\frac{5}{2}}t + c$$

$$3) \text{ 令 } p = tx \quad \text{则} \quad x = \frac{1}{\sqrt{1-3t^2}}, \quad y = \frac{t}{2(1-3t^2)} - \frac{1}{4\sqrt{3}} \ln \frac{1+\sqrt{3}t}{1-\sqrt{3}t} + c$$

$$\text{令 } x = \sqrt{1+3p^2}, \quad y = \frac{p}{2}\sqrt{1+3p^2} - \frac{1}{6}\sqrt{3} \arcsin h(\sqrt{3}p) + c$$

$$4) \begin{cases} y = x^2 - p^2 \\ \frac{1}{2} \ln \left(\frac{2p^2}{x^2} - \frac{p}{x} + 2 \right) + \frac{1}{\sqrt{15}} \arctan \left(\frac{4p/x - 1}{\sqrt{15}} \right) + \ln x = c \end{cases}$$

$$5) \text{ 令 } p = tx, \text{ 则 } x = \frac{4t}{1+t^3}, y = \int \frac{4t^2(4-8t^3)}{(1+t^3)^3} dt = \frac{-8}{(1+t^3)^2} + \frac{32}{3(1+t^3)} + c$$

$$6) \text{ 令 } p = e^t \text{ 则 } x = t + e^{-t}, y = e^t - t + c$$

3.

验证必要和充分条件

$$1) x+2p=0, y=xp+p^2, \text{ 消去 } p, \text{ 我们得到: } y=-x^2/4;$$

$$2) 2x+2p=0, y=2xp+p^2, \text{ 消去 } p, \text{ 我们得到: } y=-x^2;$$

$$3) p=0, (y-1)^2 p^2 = 4y/9, \text{ 消去 } p, \text{ 我们得到: } y=0;$$

$$4) x-1/p^2=0, y^2=x^2 p^2 + 1/p^2 + 2x, \text{ 消去 } p, \text{ 我们得到: } y = \pm 2\sqrt{x}$$

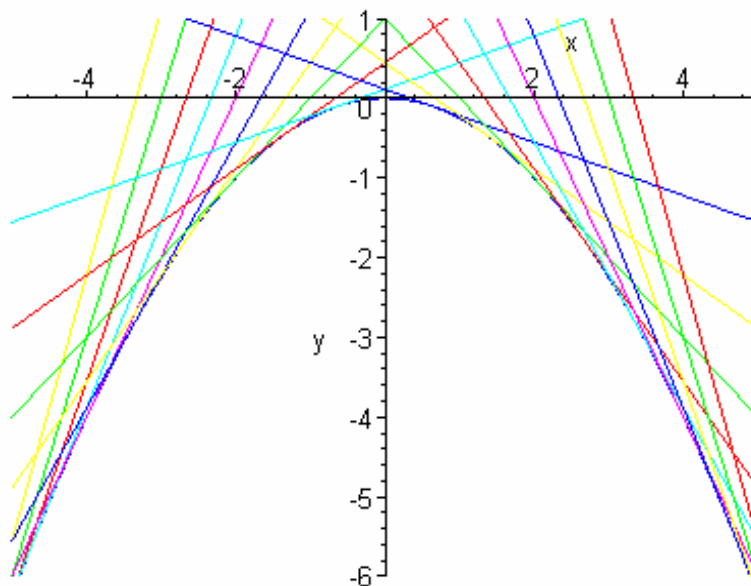
4.

$$1) \phi(x, y, c) = y - cx - c^2 = 0,$$

$$\partial\phi(x, y, c)/\partial c = -x-2c=0,$$

消去c我们得到原曲线族的包络: $y+x^2/4=0$.

```
> restart:
with(plots):
fun:=seq(subs(p=j,x*p/3+(p/3)^2),j=-10..10):
plot([fun,-x^2/4],x=-5..5,y=-6..1);
```

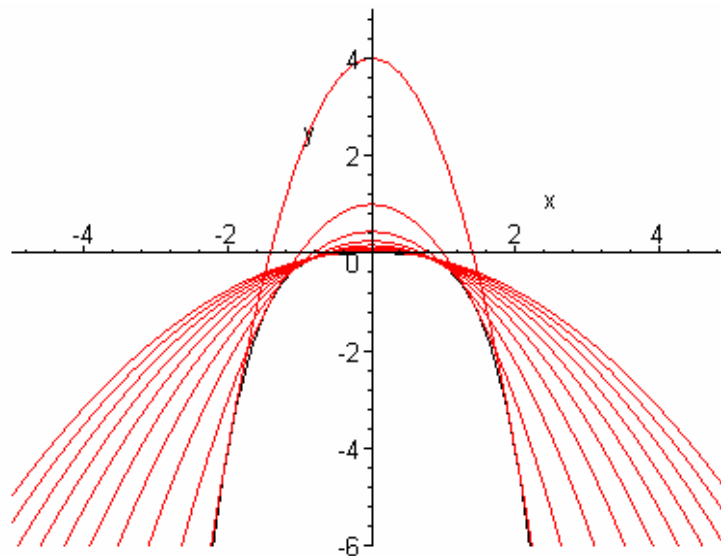


$$2) \phi(x, y, c) = c^2y + cx^2 - 1 = 0,$$

$$\partial\phi(x, y, c)/\partial c = 2cy + x^2 = 0,$$

消去c我们得到原曲线族的包络: $x^4/(4y)+1=0$.

```
> restart:
with(plots):
fun:=seq(subs(c=j/2,(1-c*x^2)/c^2),j=1..10):
plot1:=plot([fun],x=-5..5,y=-6..5,color=red):
plot2:=plot(-x^4/4,x=-5..5,y=-6..5,color=black):
display(plot1,plot2);
```

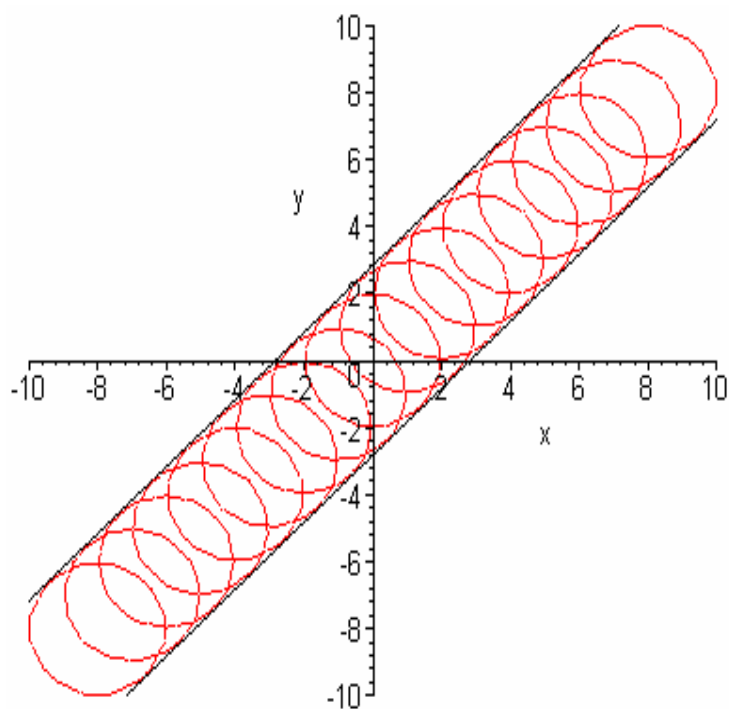


3) $\phi(x, y, c) = (x-c)^2 + (y-c)^2 = 4,$

$$\frac{\partial \phi(x, y, c)}{\partial c} = 2(c-x) + 2(c-y) = 0,$$

消去c我们得到原曲线族的包络: $(x-y)^2 = 8.$

```
> restart:
with(plots):
plot1:=seq(implicitplot((x-c)^2+(y-c)^2=4,
x=-10..10,y=-10..10,color=red),c=-8..8):
plot2:=plot([x-8^(1/2),x+8^(1/2)],
x=-10..10,y=-10..10,color=black):
display(plot1,plot2);
```



4) $\phi(x, y, c) = (x-c)^2 + y^2 = 4c,$

$\partial\phi(x, y, c)/\partial c = 2(c-x)-4=0,$

消去c我们得到原曲线族的包络: $y^2-4x-4=0.$

```
> restart:
with(plots):
plot1:=seq(implicitplot((x-c)^2+y^2=4*c,
x=-1..10,y=-8..8,color=red),c=1..8):
plot2:=plot([(4*x+4)^(1/2),-(4*x+4)^(1/2)],
x=-1..10,y=-8..8,color=black):
display(plot1,plot2);
```

