答 案 4.6

1. $\begin{cases} m\dfrac{d^2x}{dt^2}=-R_x, \\ m\dfrac{d^2y}{dt^2}=mg-R_y. \end{cases}$ 其中 R_x ,R_y 分别是阻力 R 在 x 轴,y 轴方向的向量。

初始条件为 x(0)=0,y(0)=0,$x'(0)=v_0, y'(0)=0$; 解方程得: $\begin{cases} x=\dfrac{R_x}{2m}t^2+v_0t, \\ y=\dfrac{1}{2}(g-\dfrac{R_y}{m})t^2. \end{cases}$

2. 提示：对每一个物体分别做平衡分析，建立方程，求解。

设两C和D在t时刻的位置坐标分别为x_1和x_2，坐标原点是自然状态。

$$\begin{cases} m\dfrac{d^2x_1}{dt^2}=k(x_2-x_1)-kx_1, & x_1(0)=a, \quad x_1'(0)=0, \\ m\dfrac{d^2x_2}{dt^2}=-k(x_2-x_1), & x_2(0)=b, \quad x_2'(0)=0 \end{cases}$$

引入新变量，化为一阶方程组，求解，如 a=b=10 时

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 restart:
 m:=1;k:=1;;
eq1:=diff(y1(t),t)=y2(t);
eq2:=diff(y2(t),t)=-(k+k)/m*y1(t)+k/m*y3(t);
eq3:=diff(y3(t),t)=y4(t);
eq4:=diff(y4(t),t)=k/m*y1(t)-k/m*y3(t);
dsolve({eq1,eq2,eq3,eq4,y1(0)=10,y2(0)=0,y3(0)=10,y4(0)=0
},{y1(t),y2(t),y3(t),y4(t)});
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$$m := 1$$

$$k := 1$$

$$eq1 := \frac{\partial}{\partial t}\mathrm{y1}(t)=\mathrm{y2}(t)$$

$$eq2 := \frac{\partial}{\partial t}\mathrm{y2}(t)=-2\,\mathrm{y1}(t)+\mathrm{y3}(t)$$

$$eq3 := \frac{\partial}{\partial t}\mathrm{y3}(t)=\mathrm{y4}(t)$$

$$eq4 := \frac{\partial}{\partial t}\mathrm{y4}(t)=\mathrm{y1}(t)-\mathrm{y3}(t)$$

$$\{\mathrm{y1}(t)=5\cos\left(\frac{1}{2}t\sqrt{5}-\frac{1}{2}t\right)+\sqrt{5}\cos\left(\frac{1}{2}t\sqrt{5}-\frac{1}{2}t\right)-\sqrt{5}\cos\left(\frac{1}{2}t+\frac{1}{2}t\sqrt{5}\right)$$
$$+5\cos\left(\frac{1}{2}t+\frac{1}{2}t\sqrt{5}\right), \mathrm{y4}(t)=-\sqrt{5}\sin\left(\frac{1}{2}t\sqrt{5}-\frac{1}{2}t\right)-5\sin\left(\frac{1}{2}t\sqrt{5}-\frac{1}{2}t\right)$$

$$-\sqrt{5}\sin\left(\frac{1}{2}t+\frac{1}{2}t\sqrt{5}\right)+5\sin\left(\frac{1}{2}t+\frac{1}{2}t\sqrt{5}\right),$$

$$\mathrm{y2}(t)=-2\sqrt{5}\sin\left(\frac{1}{2}t\sqrt{5}-\frac{1}{2}t\right)-2\sqrt{5}\sin\left(\frac{1}{2}t+\frac{1}{2}t\sqrt{5}\right),\ \mathrm{y3}(t)=$$

$$5\cos\left(\frac{1}{2}t\sqrt{5}-\frac{1}{2}t\right)+3\sqrt{5}\cos\left(\frac{1}{2}t\sqrt{5}-\frac{1}{2}t\right)-3\sqrt{5}\cos\left(\frac{1}{2}t+\frac{1}{2}t\sqrt{5}\right)$$

$$+5\cos\left(\frac{1}{2}t+\frac{1}{2}t\sqrt{5}\right)\}$$

3.（1）x=80t,　y=$80\sqrt{3}t-gt^2/2$.

（2）射程:2216,　最大高度:1781,　飞行时间: 27.7 秒.

（3）2 秒末的位置是(160,257), 4 秒末的位置是(320,474)

2 秒末的速度大小为（80，118）,4 秒末的速度为（80，98）.

4. $\begin{cases} I_1=3-2e^{-5t}-e^{-20t} \\ I_2=3-4e^{-5t}+e^{-20t} \end{cases}$　稳压电流是 3 安培

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restart:with(DEtools);
eq1:=diff(i1(t),t)=-15*i1(t)+5*i2(t)+30;
eq2:=diff(i2(t),t)=10*i1(t)-10*i2(t);
dsolve({eq1,eq2,i1(0)=0,i2(0)=0},{i1(t),i2(t)});
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$$eq1 := \frac{\partial}{\partial t}\mathrm{i1}(t)=-15\,\mathrm{i1}(t)+5\,\mathrm{i2}(t)+30$$

$$eq2 := \frac{\partial}{\partial t}\mathrm{i2}(t)=10\,\mathrm{i1}(t)-10\,\mathrm{i2}(t)$$

$$\{\mathrm{i1}(t)=-\mathbf{e}^{(-20\,t)}-2\,\mathbf{e}^{(-5\,t)}+3,\ \mathrm{i2}(t)=\mathbf{e}^{(-20\,t)}-4\,\mathbf{e}^{(-5\,t)}+3\}$$

5.

设t时刻这三个容器中的含盐量分别为$x_1(t)$、$x_2(t)$和$x_3(t)$,

由题意得到方程组

$$\begin{cases} \dfrac{dx_1}{dt}=-0.05x_1+20, & x_1(0)=0 \\ \dfrac{dx_2}{dt}=0.05x_1-0.025x_2, & x_2(0)=0 \\ \dfrac{dx_3}{dt}=0.025x_2-0.02x_3, & x_2(0)=0 \end{cases}$$

求解得

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restart:with(DEtools):
eq1:=diff(x1(t),t)=-0.05*x1(t)+20;
eq2:=diff(x2(t),t)=0.05*x1(t)-0.025*x2(t);
eq3:=diff(x3(t),t)=0.025*x2(t)-0.02*x3(t);
dsolve({eq1,eq2,eq3,x1(0)=0,x2(0)=0,x3(0)=0},
  {x1(t),x2(t),x3(t)});
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$$eq1 := \frac{\partial}{\partial t}\,\mathrm{x1}(t) = -.05\,\mathrm{x1}(t) + 20$$

$$eq2 := \frac{\partial}{\partial t}\,\mathrm{x2}(t) = .05\,\mathrm{x1}(t) - .025\,\mathrm{x2}(t)$$

$$eq3 := \frac{\partial}{\partial t}\,\mathrm{x3}(t) = .025\,\mathrm{x2}(t) - .02\,\mathrm{x3}(t)$$

$$\{\,\mathrm{x2}(t) = 800 + 800\,\mathbf{e}^{(-1/20\,t)} - 1600\,\mathbf{e}^{(-1/40\,t)},$$

$$\mathrm{x3}(t) = 1000 - \frac{2000}{3}\,\mathbf{e}^{(-1/20\,t)} + 8000\,\mathbf{e}^{(-1/40\,t)} - \frac{25000}{3}\,\mathbf{e}^{(-1/50\,t)},$$

$$\mathrm{x1}(t) = 400 - 400\,\mathbf{e}^{(-1/20\,t)}\,\}$$